

PROMPTING MATHEMATICS COACH DEVELOPMENT OF MATHEMATICAL KNOWLEDGE FOR TEACHING

Andrea Knapp
The University of Georgia
akknapp@uga.edu

This teacher development experiment examined the development of mathematical knowledge for teaching (MKT) of three coaches of mathematics teachers (Hill, Rowan, & Ball, 2005). Coaches were graduate students in the field of mathematics who had an interest in teaching. Coaches developed knowledge of content and teaching as they reflected and collaborated with classroom teachers to implement inquiry-oriented lessons. Coach knowledge of content and students developed through observing and interacting with students. Finally, coaches developed specialized content knowledge as they discussed perturbations from lessons with teachers.

Background

The field of professional development coaching currently enjoys steady growth in mathematics education as schools search for effective ways to support the learning of in-service teachers. Although coaching is gaining popularity as a means of professional development, its forms and effectiveness of implementation vary from context to context (Olson & Barrett, 2004). For example, due to the difficulty in recruiting highly qualified coaches from the field of mathematics education, some school districts have looked to hiring their best mathematics teachers as coaches. As a result, districts rob students of qualified teachers and position the newly hired coaches to find their own way in supporting teachers. Thus, finding qualified, cost-effective coaches remains a difficulty for schools.

One alternative model of coaching matches graduate students in the fields of science, technology, engineering, and mathematics (STEM) as content specialist coaches with K-12 classroom teachers. The National Science Foundation (NSF) has established the Graduate Fellows in K-12 Education (GK-12) program that provided the context for this study. The GK-12 program places Graduate Fellows into K-12 classrooms as *collaborative coaches* (Olson & Barrett, 2004). The partnership is designed as a mutual professional development opportunity for both the coaches and the classroom teachers. Teachers develop content knowledge related to the subject matter they teach. (See Knapp, Barrett, & Kaufmann (2007) for teachers' development of mathematical knowledge for teaching through this model.) The teachers voluntarily participate in the partnership. The mathematics graduate coaches may or may not have prior teaching experience, but they have had summer and bi-weekly training on topics such as the National Council of Teachers of Mathematics (NCTM) Standards (2000) and social constructivism. Graduate coaches collaborate with practicing teachers and other graduate coaches on planning and delivering standards-based lessons. Coaches generally model lessons for teachers with the teachers' own students. On some occasions, however, the teacher leads the lesson with the coach's assistance. Most often, teachers and graduate coaches work in pairs, although at times teachers request the assistance of multiple coaches. NSF expects that graduate coaches will develop useful teaching abilities through the program that will prepare them for faculty positions that involve teaching. In order to examine the viability of the partnership, this study examines the impact of the coaching relationship on the teaching abilities of the *coaches*. More specifically, I investigate coaches' development of mathematical knowledge for teaching, which is the

mathematical knowledge and habits of mind needed for *teaching* mathematics (Hill et al., 2005). In addition, I seek to ascertain aspects of the coach-teacher relationship that might lead to coach development. Thus I ask, “In what ways do graduate coaches develop mathematical knowledge for teaching as they engage in collaborative coaching with classroom teachers?” For the remainder of this paper, I refer to graduate coaches as *Coaches*, K-12 classroom teachers as *Teachers*, and K-12 students as *students*.

Theoretical Framework

The construct of *mathematical knowledge for teaching* (MKT) has been linked to student achievement, and thus provided a framework for analysing the Coaches’ development in this study (Hill et al., 2005). MKT includes these six elements: common content knowledge; specialized content knowledge (SCK) needed specifically for the mathematics classroom; knowledge of content and students (KCS) which is knowledge of how students learn mathematics; knowledge of content and teaching (KCT) which includes knowing the best representations for teaching mathematics; knowledge of curriculum; and knowledge at the mathematical horizon. This study focused on KCT, KCS, and SCK (See Table 1) (Hill et al.).

Table 1
Components of Mathematical Knowledge for Teaching

Subject Matter Knowledge	Pedagogical Content Knowledge
Common Content Knowledge (CCK)	Knowledge of Content and Students (KCS)
Specialized Content Knowledge (SCK)	Knowledge of Content and Teaching (KCT)
Knowledge at the Mathematical Horizon	Knowledge of Curriculum

For a theoretical framework, the emergent perspective appeared suited to this study because mathematical knowledge for teaching (MKT) is related to the construct of classroom social norms as outlined in the emergent perspective (Ball, 2003; Cobb & Yackel, 2004). The emergent perspective takes the social aspects of learning and the individual psychological aspects to be reflexively related. In this study, I investigated the individual and social construction of MKT of Coaches as they collaborated with teachers.

Methodology

I chose to employ qualitative, multi-tiered teacher development experiment (TDE) methodology because the goal of a TDE is to generate models for teachers’ mathematical and pedagogical development, closely matching our research aims for the collaborative coaches (Lesh & Kelly, 2000; Presmeg & Barrett, 2003). The methods for this qualitative teacher development experiment involved year-long case studies of three mathematics coaches: Melvin, Dave, and Marsha. I also conducted case studies on four Teachers, but I do not report on the Teacher development in this paper. Melvin had four years of teaching experience at the secondary level before pursuing his graduate degree in mathematics. Dave had two years of teaching experience, and Marsha had no former teaching experience or preparation.

Data sources for Melvin’s development of MKT included 87 written pre/post lesson reflections, transcripts of two audiotaped interviews, 4 video-taped lessons, and transcripts of 9 audiotaped pre and post planning sessions done in collaboration with a 6-8th grade mathematics teacher, Mrs. Gerber. Data sources for Dave included 74 written pre/post reflections, transcripts of two interviews, 9 video-taped lessons, and transcripts of 7 audiotaped planning sessions with

Teachers. Data sources for Marsha included 87 written pre/post reflections, transcripts of two interviews, 12 video-taped lessons, and transcripts of 11 audiotaped planning sessions with Teachers. Pre lesson reflections asked Coaches to describe the lessons that they would teach and to explain how students would be expected to invent knowledge and think through the content. Coaches were also asked to predict areas that would be difficult for students to understand. Post lesson reflections required Coaches to describe how the lesson went and whether students understood the content. These data sources were analysed for development in mathematical knowledge for teaching with regard to SCK, KCS, and KCT.

In order to analyse the data, SCK, KCS, and KCT were broken down into 17 codes relating to different aspects of each construct. A question accompanied each code in order to highlight ways that MKT development might occur. Questions came from the elements of the work of teaching elaborated by Hill et al. (2005) and from salient aspects of the pilot study. For examples of the codes and accompanying questions, see Table 2. Three transcripts were analysed by both the researcher and Melvin, Dave, and Marsha respectively until an interrater reliability of 80% was reached. After this, the researcher coded the rest of the transcripts. Each time a portion of transcript was coded as SCK, KCS, or KCT, the accompanying question was answered based on the data. Ways in which these elements developed were then categorized and tabulated.

Table 2

Analysis codes and questions

Category	Analysis Question (Code)
KCT	How did the lesson study environment affect the Teachers'/Coaches' instructional choices and use of curriculum? (KCT1)
KCT	How did the Teacher/Coach encourage student construction of knowledge? (KCT4)
KCT	How did the Teacher/Coach provide explanations, examples, or counterexamples? (KCT7)
KCT	Did the Teacher/Coach ask students to justify their reasoning? (KCT6)
KCS	How does the Teacher/Coach notice students' knowledge/reasoning/thinking as they engaged in lesson study? (KCS12)
KCS	How does lesson study help Teachers/Coaches question their students [not as an instructional tool but to learn about students' thinking]? (KCS13)
KCS	How does lesson study help Teachers/Coaches see/hear student misconceptions? (KCS14)
SCK	How did mathematical discourse between Teachers and Coaches foster reasoning? (SCK17)

Results and Discussion

I coded Coach reflections for knowledge of content and students (KCS), knowledge of content and teaching (KCT), and specialized content knowledge (SCK). This meant that I coded quotes which I felt indicated that the Coach developed or had an opportunity to develop these elements of teaching. For example, if a Coach noted changes he would make to a lesson after teaching it, I coded an opportunity to develop KCT. I avoided counting quotes which showed teaching knowledge that the Coach possessed prior to the lesson. In addition to reporting results from reflections in this section, I also report codes from transcripts of audio taped reflections

with teachers, videotapes of lessons, and interviews. The final tallies revealed 91 expressions of developing KCT, 41 expressions of developing KCS, and 14 instances of developing SCK. In addition, Coaches reported learning general pedagogy.

After coding the data, I looked back at each code, and identified *how* that type of knowledge developed. To focus on the primary ways in which Coaches developed, I eliminated all ways that were expressed less than ten times. Some ways I collapsed together into a single category. The compression phase of data analysis revealed five primary ways that mathematical knowledge for teaching developed for the Coaches. I list the five primary ways in Table 3, and I provide evidence for the ways in the sections following the table.

Table 3

Primary Ways in which Coaches Developed Mathematical Knowledge for Teaching

Frequency	Way
KCT (46)	
17	Reflecting about changes and things to keep from a lesson taught
11	Having students construct their own knowledge and make discoveries and conclusions; Teaching for conceptual understanding
18	Reflecting on lesson with teacher; Considering perturbations or misconceptions prompting revision; Collaborating with Teacher in planning, with both sets of expertise working together; Repeating lesson or teaching strategy
KCS (32)	
32	Observing , listening, and interacting with students during lesson; seeing what's hard for them and misconceptions; seeing level of material students could handle, construct, or conjecture (raised expectations)
SCK (10)	
10	Finding an application to model the content; Discussing perturbations with Teachers or other Coaches; Testing a student conjecture

Coach Development of Knowledge of Content and Teaching

The primary ways in which Coach mathematical knowledge for teaching developed focused on knowledge of content and teaching (KCT) and knowledge of content and students (KCS). The first primary way Coaches developed KCT was in reflecting about lessons they had taught and deciding to either retain or change elements of the lessons. For example, Marsha stated the following in a reflection:

The kids really had a lot of fun with this game, and it really was simple in terms of materials and set up. They practiced a lot of multiplication problems and were really checking each others work since they wanted to win the cards that round. It also helped them to think about strategies of what is going to give you a big number when you multiply, which I think helps them to develop their estimating and telling if an answer is reasonable.

She learned a classroom activity to support students' estimating strategies (KCT). This finding substantiates Mumba et al. (2003) who found Coaches to have Technocratic-oriented reflections. In other words, they sought solutions to improve their teaching. Furthermore, like Mumba et al., the nature of the Coach reflections were descriptive, dialogical, and critical. Dialogical referred to reflecting with teachers during and after lessons as well as reflecting with other Coaches before and after lessons. Critical referred to expressing dissatisfaction with lessons and suggesting alternatives. The second way Coaches developed KCT followed from the first, in that

Coaches often chose to change lessons towards inquiry. In other words, they valued to a greater degree students making their own discoveries and conclusions, individual construction of knowledge, and teaching for conceptual understanding. The third way KCT developed was similar to the first and second, but allowed for collaboration with Teachers. Coaches collaborated with Teachers on planning lessons, in which both sets of expertise went into the lesson design. The teacher contributed knowledge of her individual class and pedagogy. The Coach contributed content knowledge and knowledge about inquiry-based instruction. After lessons, Coaches and Teachers as collaborative partners discussed perturbations relating to student misconceptions, pedagogy, or technology. The Coaches and Teachers would then revise and sometimes repeat lessons based on their discussion of the issues. Finally, Coaches would at times repeat the cycle of revision, perhaps with another Teacher or another Coach.

Coach Development of Knowledge of Content and Students

Knowledge of content and students (KCS) primarily developed during lessons as Coaches observed, listened to, and interacted with students. Coaches learned what concepts were difficult for students to understand as well as misconceptions that students possessed. For example, all three Coaches independently encountered trapezoid as a challenging concept in middle grades classrooms. Dave and Marsha, independently in different lessons and different schools, found that students' conceptions of 'trapezoid' are often of an isosceles trapezoid where both legs are the same length. The Coaches then challenged this misconception by presenting them with right trapezoids, and discussing with them the properties of a trapezoid. Marsha used geoboards as a teaching tool and Dave used Geometer's Sketchpad to address the misconceptions. Melvin addressed the misconception during a class discussion. In a post lesson reflection he wrote,

The best discussion I felt like came from discussing the trapezoids, as the students felt there was only one possible trapezoid, the picture we frequently see in books of an isosceles trapezoid. After determining the definition of a trapezoid to be one set of parallel sides, the students debated me as to whether or not a right trapezoid was really a trapezoid. This discussion really brought out the misconceptions created by always using the same figure to describe a family of figures and was a good chance to discuss what properties are necessary to classify a quadrilateral.

Thus, the Coach learned that students hold limited concept images of figures, developing knowledge of content and students (Vinner & Hershkowitz, 1980).

In addition to learning about student misconceptions, Coaches learned about the level of material the students could handle. Moreover, Coaches learned how students created conjectures and constructed knowledge. At times, Coaches' expectations were raised by observing students' productions in an open-ended environment. For example, in an open-ended lesson in which students were to conjecture about quadrilateral properties using Geometer's Sketchpad as a tool, the Coach, Melvin, expected students to come up with perhaps four or five conjectures. He and the teacher were amazed when the students produced twenty conjectures, some of which can be seen in the following portion of transcript:

Student: 2 sets of parallel lines.

Melvin: 2 sets of parallel lines. What do we think?

Students: Yeah, No [chorus of no's]

Melvin: Who says no? OK right there, you say no? Alright, so far she's the winner. She says that's not the case, but let's go back and let's look at this, OK. On your screen you might have parallel sides, but remember what a property is. A property is something that's true

for every quadrilateral. So let's look over here. Do these have parallel sides? [motioning to a student screen and dragging] Are they parallel now?

Melvin: Look at mine up here [referring to the projected Sketchpad screen]. What I'm going to do is I'm going to start dragging this thing around, right? I can drag this around. Are those parallel? [chorus of no's] So it doesn't. Even though I can take these and I can make them parallel, and according to that observation right there, they are parallel. So I would agree with the young lady. However, we need to be true for all of the quadrilaterals, and so that isn't going to be one for all of them.

Student: All have four letters.

Student: Four segment lines.

Melvin pointed out that "segment lines" was contradictory and wrote, "Four segments." The transcript continues.

Student: They're all enclosed areas, I mean, inside it's all enclosed.

Melvin: OK, I'm going to tie all that up with one word – polygon, OK. Enclosed.

In the audiotaped post reflection between the Coach and the teacher, the Coach said,

Yeah, I felt, I was very happy with their creativity, and their coming up with the properties. They all were involved. One thing we talked about might happen is that they'll just stare at me, and look at me. That didn't happen at ALL. So, so by that I was taken a little bit aback and was kind of enthused by it. Um, was maybe a little bit overwhelmed..."

Melvin learned that students are curious and eager to conjecture, and he also learned that student conjectures could appear very different from properties listed in a textbook. Thus, at times, the results of inquiry-based lessons took Coaches by surprise; knowledge of content and teaching had spawned knowledge of content and students.

Coach Development of Specialized Content Knowledge

The last area of Coach development related to specialized content knowledge. Although this type of development occurred less frequently than KCT and KCS, it surfaced through rich discussion as can be seen in the following example. Melvin, like Dave and Marsha, encountered trapezoids when he and a teacher developed a quadrilateral taxonomy in which a trapezoid was defined as having exactly one pair of parallel sides (Battista, 1998). During a debriefing session, the following discussion ensued between Melvin and the 7th grade Teacher, Mrs. Gerber.

Melvin: But see now the tricky thing, actually, I'm learning a lot, Mrs. Gerber, because having done this kite, now I see the relationship a kite... rhombus,

Mrs. Gerber: OK

Melvin: Every rhombus is a kite, by my, by our definition, and see this is the thing. Um, I'm finding out that that definitions vary from book to book. (That's true.) Do you guys have, do you guys talk about kite in your book?

Mrs. Gerber: No, we don't talk about it. No, we don't.

Melvin: I'm finding a lot of differences in the definition of trapezoid, a lot of differences in the definition of kite.

Mrs. Gerber: What are you finding in the differences in trapezoid?

Melvin: Some books just say that it's got um at least two [one] pairs of opposite sides [parallel], so that would mean a parallelogram is a trapezoid as well.

Mrs. Gerber: OK

Melvin: But it doesn't have to have all, it doesn't have to have all, it doesn't have to have two pairs of opposite sides parallel. It just has to have at least two opposite sides that are

parallel, so that gives it a little bit of more flavor, so every parallelogram is a trapezoid, and every trapezoid is a quadrilateral, it would fit in that flow.

Mrs. Gerber: Oh, OK, now I've never seen that either.

Melvin: Yeah, actually, I like that better, if we're going to make a note on how things are going to be done next year, I would suggest doing that.

Mrs. Gerber: Well, you know, that that make the trapezoid not such an odd guy out.

In his final interview, Melvin stated,

I will be honest with you, I was growing in my understanding as well, right with her [Mrs. Gerber]. I mean, because I, we had presented trapezoids as only one pair of opposite parallel sides as opposed to at least [one pair of parallel sides]. And that change makes quite a difference...

Thus, Melvin developed specialized content knowledge (SCK) about definitions of a trapezoid as he delved into the topic of quadrilateral relationships with the teacher. In another planning session, Melvin and the Teacher wrestled with adding arrows to their taxonomy to denote generality (Battista, 1998). For example, they showed that a square is always a rectangle with a down arrow, but that a rectangle is sometimes a square with an up arrow (See Knapp et al., 2007). They decided to give the same task to students. The collaboration between Melvin and Mrs. Gerber inspired a didactic problem situation, and thereby provided for the development of knowledge of content and teaching (KCT).

Coaches' Path to Mathematical Knowledge for Teaching

In reflecting about the ways in which Coaches developed mathematical knowledge for teaching, I summarize the development with the following cycle. In the *Coaches' Path to Mathematical Knowledge for Teaching*, Coaches develop specialized content knowledge (SCK) as they research the topic for a lesson. As they search for curriculum and collaboratively plan for instruction with Teachers and other Coaches, they develop specialized content knowledge (SCK) and knowledge of content and teaching (KCT). During the lesson, Coaches develop knowledge of content and students (KCS) through observation of and discourse with students. Finally, in the reflection/debriefing phase, Coaches develop KCT, KCS, and SCK as they consider perturbations from the lesson with the Teacher or another Coach and reflect on the lesson (Hart, Najee-Ullah, & Schultz, 2004). Finally, the lesson may be repeated at another school with another teacher, and the cycle continues. The debriefing phase is perhaps the most valuable aspect of the cycle because both mathematical and pedagogical issues get discussed and intertwined. For example, the definition of a trapezoid discussion between Melvin and Mrs. Gerber occurred during a debriefing meeting. Also during the debriefing sessions, the lessons are fine-tuned based on the knowledge of content and students gained from the teaching phase. It is important to note that not all Coaches follow this path for all lessons. Rather, it is when and to what degree Coaches follow this path or elements of it that they appear to develop mathematical knowledge for teaching.

In conclusion, Graduate Coaches developed MKT through collaborative coaching with teachers. In particular, development occurred during collaborative planning and subsequent debriefing with Teachers. This aspect of the Coaching relationship is critical and should not be shortchanged. This research implies that the coaching relationship can serve as a form of professional development for both Teachers and Coaches (Knapp, et al., 2007). Thus, I recommend that universities involved in the preparation and professional development of STEM

professors or K-12 teachers consider offering coursework, student teaching assignments, or assistantships which involve coaching of K-12 teachers.

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Different methods of teaching mathematics have been proposed by different educators and the knowledge of these methods may help in working out a better teaching strategy. It is not appropriate for a teacher to commit to one particular method. A teacher should adopt a teaching approach after considering the nature of the children, their interests and maturity and the resources available. Effective math teaching supports students as they grapple with mathematical ideas and relationships. Allow them to discover what works and experience setbacks along the way as they adopt a growth mindset about mathematics. 14. Build excitement and reward progress. Students—especially those who haven't experienced success—can have negative attitudes about math. Consider having students earn points and receive certificates, stickers, badges, or trophies as they progress. Weekly announcements and assemblies that celebrate the top players and teams can be really inspiring for students. 15. Having the Knowledge for teaching mathematics. 57. Common content knowledge for mathematics. 58. Approaches to teacher development that sustain teacher learning. 59. Considering systematic planning for teacher learning. This review of research into aspects of mathematics teaching focuses on issues relevant to Australian mathematics teachers, to those who support them, and also to those who make policy decisions about mathematics teaching. It was motivated by and draws on the proceedings of the highly successful Australian Council for Educational Research Council (ACER) conference titled Teaching Mathematics? Make it count: What research tells us about effective mathematics teaching and learning, held in Melbourne in August 2010.