Better learning of geometry with computer

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ABSTRACT
The purpose of the research is to determine whether the use of computer assisted instruction and classroom intervention will significantly improve the primary school learners ' geometric understanding of concepts. The research focuses on the learners' ability to engage in computer generated instruction for learning outcome 3- SPACE AND SHAPE. The research involves grade 7 learners with pre- and posttests being administered to ascertain whether learners can benefit from the use of computer generated instruction.

Geometry is the study of spatial relationships. It is connected to every strand in the mathematics curriculum and to a multitude of situations in real life. Geometric figures and relationships have played an important role in society's sense of what is aesthetically pleasing. From the Greek discovery and architectural use of the golden ratio to M. C. Escher's use of tessellations to produce some of the world's most recognizable works of art, geometry and the visual arts have had strong connections. Well-constructed diagrams allow us to apply knowledge of geometry, geometric reasoning, and intuition to arithmetic and algebra problems. The use of a rectangular array to model the multiplication of two quantities, for instance, has long been known as an effective strategy to aid in the visualization of the operation of multiplication. Other mathematical concepts which run very deeply through modern mathematics and technology, such as symmetry, are most easily introduced in a geometric context. Whether one is designing an electronic circuit board, a building, a dress, an airport, a bookshelf, or a newspaper page, an understanding of geometric principles is required.

Spatial thinking involves visual imagery processes such as recognition of shapes, transforming shapes, and seeing parts within shape configurations.Learners in early primary school begin to reason about shapes by considering certain features of the shapes as well as using their prototypical images. Spatial thinking plays a role in making sense of problems and in representing mathematics in different forms such as diagrams and graphs. A degree of spatial awareness and related meanings are essential for using manipulatives in many aspects of mathematics. For example, separateness is necessary for counting and yet collections can assist with establishing composite units.

Spatial sense is an intuitive feel for shape and space. It involves the concepts of traditional geometry, including an ability to recognize, visualize, represent, and transform geometric shapes. It also involves other, less formal ways of looking at two- and three-dimensional space, such as paper-folding, transformations, tessellations, and projections. Geometry is all around us in art, nature, and the things we make. Learners of geometry can apply their spatial sense and knowledge of the properties of shapes and space to the real world. Traditionally, elementary school geometry instruction has focused on the categorization of shapes and at the secondary level, it has been taught as the prime example of a formal deductive system. By virtue of living in a three-dimensional world children enter school with a remarkable amount of intuitive geometric knowledge. In early elementary school, a rich, qualitative, hands-on study of geometric objects helps young children develop spatial sense and a strong intuitive grasp of geometric properties and relationships. Eventually they develop a comfortable vocabulary.

Rationale for teaching and learning *Spatial Sense* in the Primary School
In our schools in South Africa we have five learning outcomes in mathematics and Learning Outcome 3 is dedicated to Space and Shape (Geometry). From grades R to grades 7 the Revised National Curriculum Statements attest to the importance of 2 and 3 dimensional shapes."*The learner will be able to describe and represent characteristics and relationships between two-dimensional shapes and three-dimensional objects in a variety of orientations and positions."*
Learning Outcome focus
The study of space and shape enables the learner to:

- develop the ability to visualise, interpret, calculate relevant values, reason and justify; and
- interpret, understand, classify, appreciate and describe the world through two-dimensional shapes and three-dimensional objects, their location, movement and relationships.

The learner should gain these skills from experiences with concrete objects, through drawing and construction, and in the abstract justification of spatial relationships. The learner's experience of space and shape moves from recognition and simple description to classification and more detailed description of features and properties of two-dimensional shapes and three-dimensional objects. Learners should be given opportunities to:

- draw two-dimensional shapes and make models of three-dimensional objects; and
- describe location, transformations and symmetry.

Standard 3 of the NCTM Draft "Standards 2000" Document (1998) suggests that mathematics instruction programmes should pay attention to geometry and spatial sense so that all learners, among other things, "use visualisation and spatial reasoning to solve problems both within and outside of mathematics". One of the six strands in the Western Australian Curriculum focuses on "the visualisation, analysis, representation and interpretation of shapes and objects in space" (Student Outcome Statements, Working Edition, 1994). But what is the value of the study of space and the development of spatial sense as suggested in the above curricula? Learners are surrounded by spatial settings and the ability to perceive spatial relations is regarded as important for **everyday interaction in space**. For example, Smit (1998) stresses the importance of these skills: "*Without spatial sense it would be difficult to exist in this world – we would not be able to communicate about position, relationships between objects, giving and receiving directions or imagine changes taking place regarding the changes in position and size of shapes."* This quote from the work of Van Niekerk (1995) suggests the value of spatial sense for the study of formal **geometry**: "*The Geometry curriculum for the primary school should start with the real world of the child. The intuitive notions that children reveal when exposed to spatial situations should be capitalised on (van Hiele, 1982). Once the child has experienced these situations he/she must be able to reflect on them. It is only possible to reflect if there is an underlying relationship between the experiences he/she is exposed to….Geometry does not start with the formulation of definitions and theorems. It already starts when the child has to orientate him/herself in the everyday surroundings. This familiarisation with the physical environment will eventually lead to more experiences that pave the way for developing these definitions and theorems (Freudenthal, 1991)"*

Furthermore, research has shown that learners are inappropriately prepared for the formal geometry demanded by the curriculum. According to the van Hiele theory, a learner needs to be on the ordering van Hiele level to cope meaningfully with the an axiomatic system. Research in South Africa (De Villiers & Njisane, 1987; Smith, 1987) and elsewhere (Senk, 1989; Usiskin, 1982; Shaughnessy & Burger, 1985) has shown that many school learners are only on the van Hiele visual or analysis levels. As a result learners cannot find a meaningful interpretation of the activities required at high school and resort to memorisation.

Different ways in which people interact in physical space may be distinguished. These include:
1. Observing spatial objects in a discriminating way, that is, two and three dimensional figures.
2. Determining distances, elevations, area and volumes.
3. Designing spatial objects and configurations, for example, gardens, furniture arrangements, furniture, buildings and artistic designs.
4. Representing spatial configurations with plane drawings.
5. Interpreting plane representations of spatial configurations.

Traditional school geometry in South Africa has attempted to address the first three aspects, but is singularly lean on the rich domain of geometrical ideas pertaining to aspects (4) and (5). Considering that so much of our interaction in physical space involves dealing with two dimensional representations of this space. Some work on simple projections will not only strongly enrich the utilitarian value of school geometry, but will extend the content beyond the domain of describing the properties of plane figures.

Learners of all ages should recognize and be aware of the presence of geometry in nature, in art, and in human-built structures. They should realize that geometry and geometric applications are all around them and, through study of those applications, come to better understand and appreciate the role of geometry in life. Carpenters use triangles for structural support, scientists use geometric models of molecules to provide clues to understanding their chemical and physical properties, and merchants use traffic-flow diagrams to plan the placement of their stock and special displays. These and many, many more examples should leave no doubt in learners' minds as to the importance of the study of geometry.

Experiential education is based on the idea that active involvement enhances learners' learning. Applying this idea to mathematics is difficult, in part, because mathematics is so 'abstract'. One way of bringing experience to bear on learners' mathematical understanding, however, is the use of manipulatives. Manipulatives are small, usually very ordinary objects that can be touched and moved by learners to introduce or reinforce a mathematical concept. Manipulatives come in a variety of forms, from inexpensive, simple buttons or empty spools of thread to tangrams and pattern blocks. Typically, it has been the primary grades' educators who have generally accepted the importance of manipulatives. "Both Pestalozzi, in the 19th century, and Montessori, in the early 20th century, advocated the active involvement of children in the learning process.

THE CONTEXT

Dissatisfaction with the secondary school geometry curriculum and poor performance in geometry has been the topic of many discussions over the past decade or two. During 1997 the Geometry Working Group of a South African Mathematics Non Governmental Organization attempted to re-conceptualise the teaching and learning of Geometry (Bennie, 1998). Many educators teach 3 dimensional shapes using available resources namely the text book and the chalkboard. Worksheets of 3 dimensional shapes are displayed as a 2 dimensional shape. The apparent lack of depth of teaching 3 dimensional shapes has a lasting impression on the minds of the learners. The learner's perception of 3 dimensional shapes is that of it being a 2 dimensional shape. The approach of the educator in teaching 3 dimensional figures using 2 dimensional medium has a negative impact on the geometrical understanding of learners in the later grades.

The last systemic evaluation conducted by the Department of Education found that learners were still grappling with the understanding of geometric concepts. Also the 2003 TIMMS report showed that the learners from South Africa did extremely poorly in geometry.It was felt that if the process is to be continued, and if changes to the curriculum are to be proposed, a better understanding of the geometric thinking of learners is required. The Van Hiele model of thinking is useful in providing a framework in which to work as well as providing good guidelines for the designing of geometry activities for use in mathematics classes.

Primary school educators generally tend to spend the minimum amount of instruction time on the teaching of geometry. When the subject is taught it is usually done using the traditional transmission model. Learners thus have the problem of poor conceptual understanding in the higher standards when a deeper knowledge of geometric concepts is expected/presupposed. This view is corroborated by Pegg & Davey (1998) when they express the opinion that "…there is increasing evidence that many learners in the middle years of schooling have severe misconceptions concerning a number of important geometric ideas."

Computers can be used to enhance a learner’s knowledge of mathematics, focusing on what can be done above and beyond with pencil and paper alone (Pea, 1986). Using computers as cognitive tools to assist learners in learning powerful mathematics that they could have approached without the technology should be a key goal for research and development—not only learning the same mathematics better, stronger, faster, but also learning fundamentally different mathematics in the process (Jonassen & Reeves, 1996; Pea, 1986). There are a number of potential benefits of using the computer as a tool for instruction in an educational setting. First, technological tools help to support cognitive processes by reducing the memory load of a learner and by encouraging awareness of the problem solving process. Second, tools can share the cognitive load by reducing the time that learners spend on computation. Third, the tools allow learners to engage in mathematics that would otherwise be out of reach, thereby stretching learners' opportunities. Fourth, tools support logical reasoning and hypothesis testing by allowing learners to test conjectures easily (Lajoie, 1993). Computers allow for one to record problem-solving processes.

This research aims to develop an instructional model to be implemented in an effort to improve the geometric understanding of primary school learners. The model will be work shopped with educators to evaluate its appropriateness and to assess whether it can have an impact on learners’ performance.

LITERATURE REVIEW

THE VAN HIELE THEORY

The van Hiele theory is arguably the best-known framework presently available for studying teaching and learning processes in Geometry. The theory was developed in the 1950’s when two Dutch mathematics educators, Pierre van Hiele and his wife Dina van Hiele-Geldof, did research on thought and concept development in geometry. They also investigated the role of instruction in assisting learners to acquire geometric knowledge and raise their thought levels.

Resulting from his research van Hiele developed the notion of developmental levels of thinking in geometry. Van Hiele's ideas have, according to Pegg (1992), a lot in common with Piaget's model of cognitive development in that both models ascribe the development of learner understanding to a series of levels or stages. From his research van Hiele has identified and proposed five levels of understanding through which learners must progress in their geometric thought development. Fuys et al (1988) characterized the levels as follows:
Level 0: (Recognition/Visualization). Learners identify, name, compare and operate on geometric figures on the basis of their appearance in a holistic manner.
Level 1: (Analysis). Learners analyze figures in terms of their components and discover the relationships among those components as well as derive the properties/rules of a class of shapes empirically.
Level 2: (Informal Deduction). Learners logically interrelate previously discovered properties/rules by giving or following informal arguments.
Level 3: (**Deduction**). Learners prove theorems deductively and establish interrelationships among networks of theorems.
Level 4: (Rigor). Learners establish theorems in different postulation systems and analyze/compare these systems.

This research project and intervention programme has, to a large extent, been inspired by the following factors:

- Fuys et al (1988) indicated in their research findings that attainment of Level 1 (Analysis) is a reasonable goal for children whilst at primary school.
- Two previous studies exploring the van Hiele Theory were done in South Africa. The first (Smith, 1987) used Grades 9 and 10 learners as sample whilst the second by McAuliffe (1999) concentrated on the geometric understanding of pre-service educators. There seems to be a lack of evidence of research in this field at primary school level.
- The apparent neglect of geometry teaching in the primary school and the subsequent poor performance at high school level.

The Math Cognitive Development Scale
The table below represents the current thinking on a six-level Piagetian-type scale for school mathematics. It is an amalgamation and extension of ideas of Piaget and the van Hieles. The first three levels are particularly relevant to elementary school learners.

Stage Name	Math Developments
Level 1. Piagetian and Math sensorimotor.	Infants use sensory and motor capabilities to explore. Research on very young infants suggests some innate ability to deal with small quantities such as 1, 2, and 3. As infants gain crawling or walking mobility, they can display innate spatial sense.

Level 2. Piagetian and Math preoperational.	Children begin to use symbols, such as speech. They respond to objects and events according to how they appear to be. The children are making rapid progress in receptive and generative oral language. They accommodate to the language environments they spend a lot of time in.Children learn some folk math and begin to develop an understanding of number line. They learn number words and to name the number of objects in a collection and how to count them, with the answer being the last number used in this counting process.A majority of children discover or learn “counting on” and counting on from the larger quantity as a way to speed up counting of two or more sets of objects. Children gain increasing proficiency in such counting activities. In terms of nature and nurture in mathematical development, both are of considerable importance during the preoperational stage.
Level 3. Piagetian and Math concrete operations.	Children begin to think logically. In this stage, which is characterized by 7 types of conservation: number, length, liquid, mass, weight, area, volume, intelligence is demonstrated through logical and systematic manipulation of symbols related to concrete objects. Operational thinking develops (mental actions that are reversible).While concrete objects are an important aspect of learning during this stage, children also begin to learn from words, language, and pictures/video, learning about objects that are not concretely available to them.this takes place in elementary school stage. Learning math is linked to having previously developed some knowledge of math words and concepts. However, the level of abstraction in the written and oral math language quickly surpasses a learner’s previous math experience. That is, math learning tends to proceed in an environment in which the new content materials and ideas are not strongly rooted in verbal, concrete, mental images and understanding of somewhat similar ideas that have already been acquired.
Level 4. Piagetian and Math formal operations. Van Hiele level 2: informal deduction.	Thought begins to be systematic and abstract. Intelligence is demonstrated through the logical use of symbols related to abstract concepts, problem solving, and gaining and using higher-order knowledge and skills.Math maturity supports the understanding of and proficiency in math at the level of a high school math curriculum. Beginnings of understanding of math-type arguments and proof. Piagetian and Math formal operations includes being able to recognize math aspects of problem situations in both math and non-math disciplines, convert these aspects into math problems (math modeling), and solve the resulting math problems if they are within the range of the math that one has studied. Such transfer of learning is a core aspect of Level 4
Level 5. Abstract mathematical operations. Van Hiele level 3: deduction.	Mathematical content proficiency and maturity at the level of contemporary math texts used at the senior undergraduate level in strong programs, or first year graduate level in less strong programs. Good ability to learn math through some combination of reading required texts and other math literature, listening to lectures, participating in class discussions, studying on your own, studying in groups, and so on. Solve relatively high level math problems posed by others. Pose and solve problems at the level of one’s math reading skills and knowledge. Follow the logic and arguments in mathematical proofs. Fill in details of proofs when steps are left out in textbooks and other representations of such proofs.

Level 6. Mathematician. Van Hiele level 4: rigor.	A very high level of mathematical proficiency and maturity. This includes speed, accuracy, and understanding in reading the research literature, writing research literature, and in oral communication of research-level mathematics. Pose and solve original math problems at the level of contemporary research frontiers.

METHODOLOGY

The Sample

A Pretest-Posttest model, was used to measure the effect of the intervention programme on the geometric performance. The programme was implemented at a South African urban primary school with the sample consisting of 115 English-speaking learners from a Grade 7 class.

The Instruments

The instruments used for the research consisted of a test that was administered as pretest and posttest. The test consisted of :
1. Matching 2 dimensional shapes in different orientation.
2. Identifying nets of cubes,tetrahedra and octahedron

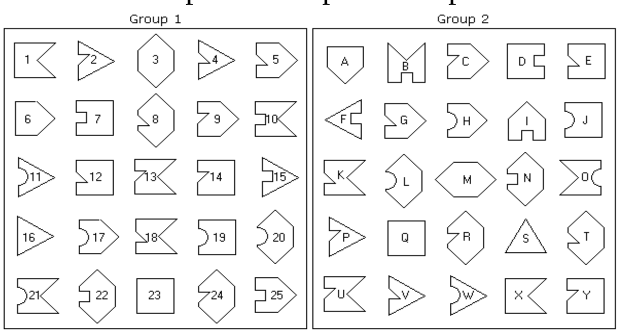
The **pretest** was administered at the commencement of the research project . This was then followed by an intervention programme. The learners were now exposed to POLY and National Library of Virtual Manipulatives where they were able to manipulate 3 dimensional shapes.

The **posttest** was administered at the conclusion of the contact session.

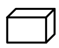
1. Shape Matching Questions

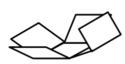
In this example, you are asked to look at two groups of simple, flat objects and find pairs that are exactly the same size and shape. Each group has about 25 small drawings of these 2-dimensional objects. The objects in the first group are labeled with numbers and are in numerical order. The objects in the second group are labeled with letters and are in random order. Each drawing in the first group is exactly the same as a drawing in the second group. The objects in the second group have been moved and some have been rotated.

1. Which shape in Group 2 corresponds to the shape in Group 1?

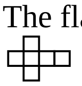


2. Nets of a cube

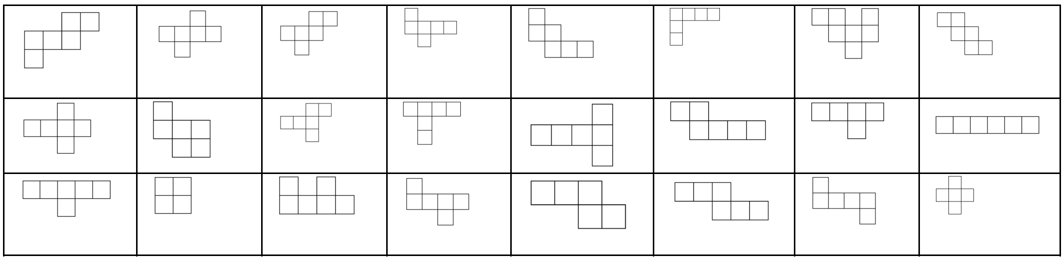
Take a cardboard box like this: 

Cut the edges of the box so that you can open it up and lie it flat: and : 

The flat box looks like this figure

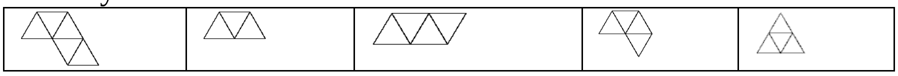


Which of the following nets will fold into a cube



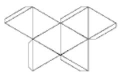
3. Nets of tetrahedra

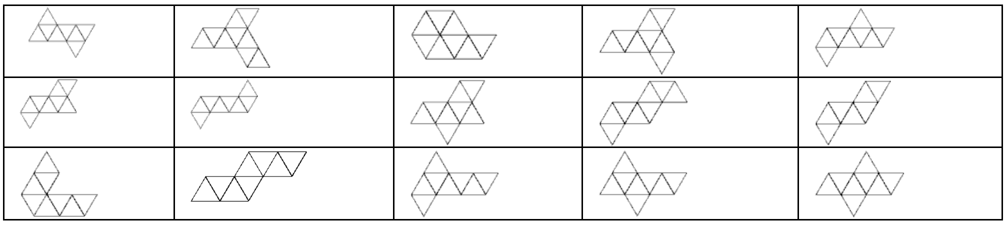
See if you can work out which nets will make a tetrahedron.



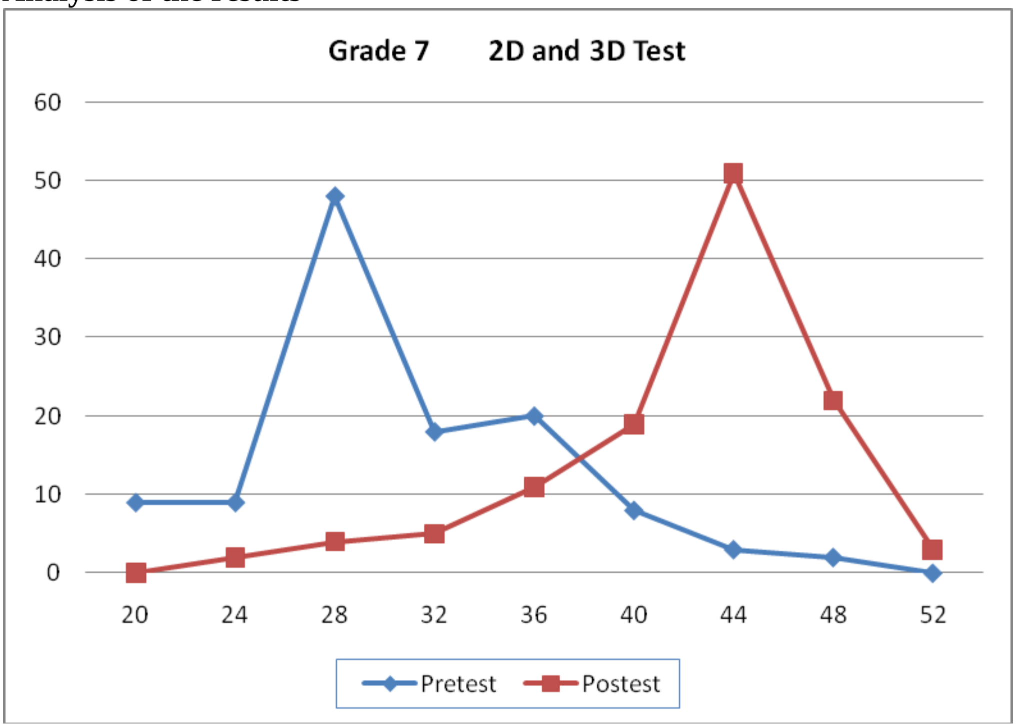
4. Nets of octahedron

Of course, for an octahedron, you must have eight triangles in the net. There are also four triangles round a point. See if you can find all the nets for an below.





Analysis of the results



DISCUSSION

The activities were designed in such a way that the learners could interact and handle all the manipulatives that were available. The activities included, for example, opportunities for learners to sort, group, draw, form new shapes. The learners seemed to have enjoyed handling the different shapes in a concrete manner before committing their ideas to paper.

CONCLUSION

Although preliminary test results obtained, as well as responses and feedback from both participating learners seem positive, the process still has some way to go. It will, of necessity, be adapted as dictated by changing circumstances and also the rapidly changing South African educational scenario. I hope that the instructional model being developed will, in some small way, contribute to a better understanding of geometric concepts by learners

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The best answers are voted up and rise to the top. Home. Questions.Â How is differential geometry (or any type of theoretical math) being used in computer science? Any research I have done on this topic leads me to some sort of applied math concept. I know that there are many pure mathematicians in physics (i.e. in string theory, studying Hamiltonionan mechanics, Quantum Field Theory and so on) and am wondering where the pure mathematicians in computer science fit in.Â I want to know the pure math concepts being used in current computer science. differential-geometry soft-question computer-science applications. share | cite | improve this question |. Free, interactive video lessons on geometry!Â Learn what angles are, and how to measure them. Right, acute, and obtuse. Learn the names for angles of all sizes. Parallel lines. Lines that never, ever cross. Perpendicular lines. Lines that cross, forming right angles. Naming angles. With these rules, you know which angle you mean. Explore the entire Geometry curriculum: angles, geometric constructions, and more. Try it free!Â IXL offers hundreds of Geometry skills to explore and learn! Not sure where to start? Go to your personalized Recommendations wall to find a skill that looks interesting, or select a skill plan that aligns to your textbook, state standards, or standardized test. IXL offers hundreds of Geometry skills to explore and learn! Not sure where to start? Go to your personalized Recommendations wall to find a skill that looks interesting, or select a skill plan that aligns to your textbook, state standards, or standardized test.