

# Inertial Mass and its Various Representations

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## Abstract

Newton introduced the concept of mass in his *Principia* and gave an intuitive explanation for what it meant. Centuries have passed and physicists as well as philosophers still argue over its meaning. Three types of mass are generally identified: inertial mass, active gravitational mass and passive gravitational mass. In addition to the question of what role mass plays in dynamical equations and why, the origin of the particular amount of matter associated with an elementary particle as a consequence of fundamental fields has long been a topic of research and discussion. In this paper, various representations of inertial mass are discussed within the framework of fundamental (either Galilean or Poincaré invariant) dynamical equations of waves and point particles. It is shown that the derived equations have mass-like and mass parameters for waves and point particles, respectively, and that the physical meaning of these parameters sheds a new light on the fundamental problem of the nature of inertial mass.

**Keywords:** Galilean and Minkowski space-time, fundamental dynamical equations, inertial mass, and mass and mass-like parameters

## 1 Introduction

The concept of mass was originally introduced by Newton [1] who wrote in his *Principia*: 'The quantity of matter is the measure of the same, arising from its density and bulk conjointly'. According to Jammer [2], a major step in interpretation of Newton's concept of mass was made by Euler in his *Mechanica*

[3], where he suggested that mass should be defined as the constant ratio between a constant force and the acceleration caused by this force. Euler's definition of mass had been widely accepted in the nineteenth century, however, later in that century, Newton's concept of force had become strongly criticized and as a result new definitions of mass independent of Newton's second law were proposed.

The first step was made by Saint-Venant [4] in 1845, when he used the principle of conservation of linear momentum to express the ratio of masses of two bodies in terms of their velocity increments after an impact. Then, Mach [5] in 1867 introduced another definition of mass that was based on two interacting particles, which otherwise were not affected by other particles in the Universe. Mach's basic idea was to define mass in terms ratios of accelerations caused by the particle's interactions. However, this implied the existence of forces whose nature was not specified. It was also pointed out that Mach's assumption of only two particles interacting with each other was superficial. Despite these objections, Mach's definition of mass had gained some popularity and become recognized [2] as "an acceptable operational definition of a theoretical construct".

Euler's and Mach's definitions of mass are based on Newton's second and third laws, respectively. Weyl [6] also proposed a definition of mass, which was based on the conservation of momentum. Both Weyl's and Mach's definitions have much in common because the conservation of momentum and the third law have the same physical content, namely, the former is the time-integrated result of the latter. A more recent discussion of these problems can be found in a series of 'Reference Frame' articles written for *Physics Today* by Wilczek [7].

There are also other definitions of mass such as that originally introduced by Hertz [8] and some based on axiomatized mechanics [9]. In his definition of mass, Hertz refers to a number of indestructible and unchangeable particles at a given point of space and at a given time, and defines mass by weight. A rather different approach is presented in axiomatized mechanics, where Newtonian inertial mass can only be determined in Galilean reference frames in which the motion of the fixed (very distant) stars must be a disjoint motion [2,9]. A different axiomatization of mechanics proposed by Schmidt [10] intended to introduce universal concept of mass. However, the approach was based on the existence of Lagrangians, which requires solving the so-called Helmholtz inverse problem [11,12].

Different concepts of mass have also been considered by Pendse [13], Carnap [14], Kamlah [15], Zanchini and Barletta [16], and others. A comprehensive review of different concepts of mass can be found in Jammer's two books [2,17], where the second book which was written more recently also includes ideas of mass developed in modern physics. The book has one chapter devoted

to relativistic mass and another dealing with the mass-energy relation. Both concepts have been recently discussed by Okun [18] and Re Fiorentin [19], who give a new re-interpretation of the concept of mass and the relativistic mass-energy relation.

Since the measure of inertia in Special Theory of Relativity (STR) is not mass of a particle but its total (kinetic and rest) energy, Okun [18] argues that relativistic mass, which depends on particle's velocity, cannot be used as the measure of inertia. He points out that in the very low velocity limit, the relativistic rest mass becomes the same as Newtonian mass and therefore STR and Newtonian mechanics are commensurate theories; see also Jammer's discussion in his chapter devoted to relativistic mass [17]. Now, Re Fiorentin [19] reached similar conclusions, however, his approach was different as he used both the Minkowski metric and the principle of least action. His main result that mass is another way of measuring energy requires the explanation of the nature of the rest-energy, for which the author refers to the Higgs mechanism.

The basic idea of the Higgs mechanism is that space is permeated by a scalar field, which is called the Higgs field, and that particles couple to this field to acquire some energy that can be interpreted as particle's mass [20]. More massive particles couple more strongly to the field. This is a very promising idea, which recently received experimental verifications [21].

In our previous work [22-25], we used the Principle of Relativity and the Principle of Analyticity to formally derive the fundamental equations of non-relativistic and relativistic mechanics of waves and particles. For the wave mechanics, we considered free and spin-zero elementary particles described by scalar wave functions. We used the extended Galilean group [26] and the Poincaré group [27] to derive the respective Schrödinger [22] and Klein-Gordon equations [24]. We demonstrated that the Schrödinger equation is the only fundamental (Galilean invariant) dynamical equation in Galilean relativity [23] and that the second-order Klein-Gordon equation is the only fundamental (Poincaré invariant) equation in space-time with the Minkowski metric [25]. Moreover, we used the same principles to derive Newton's equations of non-relativistic and relativistic point particle mechanics. In the derived fundamental equations, we encountered mass-like and mass parameters for waves and point particles, respectively.

The main objective of this paper is to demonstrate the relevance of the mass-like and mass parameters to the concepts of inertial mass discussed above, and to describe various representations of inertial mass within the framework of the fundamental (either Galilean or Poincaré invariant) theories of waves and point particles. This paper was stimulated by the two books on mass written by Jammer [2,17], and specifically by his statement that can be found in the last chapter of the second book:

"If it were possible to define the mass of a body or particle on its own in

purely kinematical terms and without any implicit reference to a unit of mass, such a definition might be expected to throw some light on the nature of mass. Such a definition, if it existed, would integrate dynamics into kinematics and eliminate the dimension  $M$  of mass in terms of length  $L$  and time  $T$ .”

It is now our purpose to show that we have already accomplished the task suggested by Jammer, and that our results do indeed shed a new light on the fundamental problem of the nature of mass.

The paper is organized as follows. In Sec. 2, we briefly describe the method used to derive invariant dynamical equations in space-time with a given metric, and also present the obtained equations. In Sec. 3, we examine the role of the mass-like and mass parameters in the fundamental theories of waves and point particles. In Sec. 4, we determine the relationships between the mass-like and mass parameters of the theories, and present various representations of inertial mass. The nature of mass is discussed in Sec. 5, and our conclusions are given in Sec. 6.

## 2 Fundamental dynamical equations

### 2.1 Basic procedure

We are interested in describing a physical object (an elementary particle or a classical point particle) by using dynamical equations, which depend upon space and time variables that are characterized by a given metric. The dynamical equations of a given metric may be derived by the procedure used in our earlier work [22-25]. Since the procedure explains the appearance of mass-like and mass parameters in the derived dynamical equations, we now briefly describe it. The basic procedure of deriving dynamical equations for free particles is as follows:

- (i) Establish a class of observers who define a physical law; for example those in isometric frames of reference.
- (ii) Decide upon the type of theoretical description to be employed; two examples are a point particle (classical) description, and a wave description. The theory may introduce new quantities, which require an additional metric to interpret the dynamical equations, such as the measure of the amplitude of a wave in wave theories.
- (iii) Employ the Principle of Relativity, which states that all observers must identify the same physical object and write down the same dynamical equations describing its space-time evolution. This could equally well be taken as the definition of a law instead of a principle. Clearly changing the class of observers could change the form and apparent nature of the laws.

(iv) Employ the Principle of Analyticity, which requires that all things that can be measured must be described by analytic functions of the space-time variables.

## 2.2 Galilean and Poincaré invariant equations

In our previous work, we considered free and spin-zero elementary particles described by scalar wave functions. To derive Schrödinger and Klein-Gordon equations, we used the extended Galilean group [26] and the Poincaré group [27], respectively, and obtained

$$\left[ i \frac{\partial}{\partial t} + \frac{1}{2M} \nabla^2 \right] \psi = 0 , \quad (1)$$

and

$$\left[ \partial^\mu \partial_\mu + \frac{\omega_0^2}{c^2} \right] \phi = 0 , \quad (2)$$

where  $\psi$  and  $\phi$  are scalar wave functions,  $M$  and  $\omega_0$  are the so-called wave mass [22] and invariant frequency [24], respectively, and  $c$  is the speed of light.

In addition to free and spin-zero elementary particles described by scalar wave functions, we also considered free classical point particles [25] and derived both non-relativistic and relativistic versions of Newton's second law of dynamics. The obtained equations can be written in the following form:

$$m \frac{dV^i}{dt} = 0 , \quad (3)$$

and

$$M_0 \frac{dU^\mu}{d\tau} = 0 , \quad (4)$$

where  $\tau$  is the proper time,  $V^i$  is the three-velocity vector with  $i = 1, 2$  and  $3$ , and  $U^\mu$  is the four-velocity vector with  $\mu = 0, 1, 2$  and  $3$ . In addition,  $m$  represents Newtonian inertial mass that is measured in  $kg$  in the SI system of units, and  $M_0$  is a derived parameter whose units are chosen here to be the same as the wave mass  $M$ .

In deriving the above dynamical equations, we encountered the need for the four parameters ( $M$ ,  $\omega_0$ ,  $m$  and  $M_0$ ) that describe the elementary particles and have the same value in all inertial frames of reference. Each of these parameters is a manifestation of inertial mass of an elementary particle, so we call  $M$  and  $\omega_0$  the mass-like parameters, and  $m$  and  $M_0$  the mass parameters; we call  $M_0$  the mass parameter despite its units, which are the same as  $M$ , because it represents mass of a point particle. Examining the invariant dynamical equations for free particles, we can offer an interpretation for the meaning of each of the invariant constants describing a free particle in the above four different dynamical equations.

### 3 Invariant mass-like and mass parameters

In the Galilean metric, the Schrödinger equation given by Eq. (1) contains one single parameter  $M$ , which is Galilean invariant and we call it the wave mass [22]. The origin of this parameter is the definition of an elementary particle. The wave vector  $\vec{k}$  and the frequency  $\omega$  are the eigen-labels by which its wave representation may be labeled in free space, and the Galilean invariant ratio of these labels upon which all inertial observers must agree  $M = k^2/2\omega$  [22,24]. Now,  $M$  may be determined independently of (Newtonian) mass  $m$ , has units derived from space and time only, and is listed for various elementary particles in Table 2 of our paper [25]. It occurs naturally and cannot be avoided in a Galilean wave description of an elementary particle.

From the dispersion relation  $\omega/k^2 = 1/2M$ , we deduce that if a particle is caused to change its state to a new value of  $k$  in a given frame of reference, then the change in  $\omega$  is proportional to  $1/M$ . The larger  $M$ , the smaller the change in the state label  $\omega$ . Thus  $M$  measures the resistance to change in frequency of the state of a free particle, a property we relate to the inertia of the particle.

In the Minkowski metric, the Klein-Gordon equation (see Eq. 2) contains a single parameter  $\omega_0$ , which is Poincaré invariant and we called it the invariant frequency in our previous paper [24,25]. The origin of this parameter is the requirement of a Poincaré invariant description of an irreducible representation of the Poincaré group. While  $\vec{k}$  and  $\omega$  must also be eigen-labels of the irreducible representations (irreps) of the Poincaré group in any inertial frame of reference, a Poincaré invariant label is the length of the eigen four-vector  $k^\mu$ , where  $k^0 = \omega/c$ . The invariant frequency may be determined independently of wave mass and Newtonian mass but it is related to them [24]. Its units are a derived quantity, depending upon units of time only.

Values of invariant frequencies for various particles are listed in Table 2 of our paper [25]. The parameter  $\omega_0$  is a measure of the inertial properties of matter, occurs naturally and cannot be avoided in a Poincaré wave description of an elementary particle. In a given frame of reference the dispersion relation  $\omega^2 = \omega_0^2 + k^2$  allows us to deduce that the greater  $\omega_0$ , the smaller the change in  $\omega$  for a given change in  $k$ . Thus  $\omega_0$  is a measure of a particle's resistance to change in frequency  $\omega$  of the state of the elementary particle in a given frame of reference, a property we relate to the inertia of the particle.

The form of the free particle dynamical equations in point particle theories is very different from that of the wave equations. The parameters remaining after setting the forces equal to zero on the RHS of Eqs (3) and (4) are  $m$  and  $M_0$ , respectively. The parameter  $m$  in Newtonian mechanics is customarily assigned a new fundamental unit of measure, the  $kg$  in the SI system of units, while  $M_0$  is a derived parameter which we have chosen to have the same units

Metric/Theory	Invariant parameter	Dispersion relation
Galilean/wave	$M$	$k^2 = 2M\omega$
Minkowski/wave	$\omega_0$	$\omega^2 - k^2 = \omega_0^2$
Galilean/particle	$m$	$p^2 = 2ME$
Minkowski/particle	$M_0$	$P^\mu P_\mu = M_0^2$

Table 1: Invariant parameters and dispersion relations for the wave and point particle theories in space-time with the Galilean and Poincaré metrics.

as wave mass  $M$ . As already shown by us [25],  $M_0$  may be related to  $m$  as well as to  $M$  and  $\omega_0$ .

The invariant mass-like and mass parameters for the wave and point particle theories given by Eqs (1) through (4) are listed in Table 1, which also contains the corresponding dispersion relations. Other local parameters for these theories, the three-vectors  $k^i$ ,  $p^i$  and  $P^i$ , and the scalars  $\omega$ ,  $E$  and  $P^0$  are given in Table 2, with  $p^i = mv^i$  and  $P^\mu = M_0 dx^\mu/d\tau$ . The three-vectors and scalars are also acceptable labels in a given frame of reference, however, they differ in value from one inertial frame of reference to another.

Since elementary particles in Nature appear to be best described by wave equations, which have parameters with derived units, the description of inertial mass by an additional fundamental measure, the kilogram, is possible but unnecessary. For elementary particles it is less accurately known than the corresponding wave mass [24,25] and thus it should not be the measure of choice. The dynamical equations for free classical point particles given by Eqs (3) and (4) have solutions independent of the mass parameters  $m$  or  $M_0$ ; the trajectories and world lines are the same for all values of these parameters.

## 4 Relationship between parameters

### 4.1 Galilean metric

Consider a free particle moving in space-time with a Galilean metric and characterized by  $k_i$ ,  $\omega$  and  $M$  in the wave description and by  $p_i$ ,  $E$  and  $m$  in the

Metric/Theory	Scalars	Vectors
Galilean/wave	$\omega$	$k_i$
Minkowski/wave	$\omega$	$k_i$
Galilean/particle	$E$	$p$
Minkowski/particle	$P_0$	$P_i$

Table 2: Frame of reference dependent labels for the wave and point particle theories in space-time with the Galilean and Poincaré metrics.

point particle description [28]. Let us assume that it is possible to arrange an interaction with a field so that both wave and particle descriptions may be employed in determining the parameters associated with the elementary particle. These conditions are described in most derivations of Ehrenfest's theorem and we assume they can be achieved for purpose of discussion here. Using the free particle parameters listed above, one observer determines the direction of travel of a wave and the other determines the direction of travel of what he assumes to be a point particle. Since it is in fact the same object their local vector parameters  $\vec{k}$  and  $\vec{p}$  must be parallel, so their magnitudes differ only by a real constant. Thus, we may write  $\lambda k_i = p_i$  and obtain from the dispersion relations

$$E = \lambda^2 \omega \frac{M}{m} . \quad (5)$$

Here  $\lambda$  is an arbitrary real constant. We note that from its definition the units of classical mass are arbitrary, i. e. changing them changes the units of force and energy but not trajectories computed from Newton's second law. On the other hand the units of wave mass are established from the choice of units of length and time. We may choose to measure  $m$  and  $M$  in the same units so that for the same particle they are equal. Then the units of  $E$  are the same as the units of  $\omega$  if we  $\lambda = 1$ , a dimensionless number.

On the other hand it is customary to interpret Eq. (5) by writing  $\lambda M = m$  and  $E = \lambda \omega$  using the experimentally determined value of  $\lambda$ , which is of course known as the Planck constant,  $\hbar$ . The wave equations given by Eqs (1) and (2) were both derived without any reference to the Planck constant and contain only the parameters  $M$  and  $\omega_0$ , both of which can be determined without reference to the Planck constant. Since the usual classical mass introduces

an unnecessary fundamental unit into physics, we prefer relating  $m$  to the wave mass  $M$  for the same particle. For elementary particles the wave mass may be determined to almost two orders of magnitude less residual error than the residual errors in the Planck constant or classical mass. Thus the wave equations without classical mass and the Planck constant are more accurate and they should be used to describe elementary particles [24].

## 4.2 Minkowski metric

The relativistic wave equation and the relativistic point particle equation are completely independent of each other, but in an appropriate limit the relativistic wave may be interpreted as a point particle [24,25]. Using the results of these papers, we relate  $k^\mu$  and  $P^\mu$  to  $\omega_0$  and  $M_0$  by using  $k^\mu k_\mu = \omega_0^2$  and  $P^\mu P_\mu = M_0^2$ . Since  $k^\mu$  and  $P^\mu$  both provide the direction of motion along the world line for the same particle under the proper experimental setup so that both theories are valid, the two four-vectors are parallel and can differ only by their lengths. Since  $M_0$  has arbitrary units, its units can be chosen so that the lengths are the same:  $\omega_0 = M_0$ .

In general, we have  $\omega_0 = M_0 c^2$  and  $M_0$  has the same units as wave mass  $M$ , a unit derived from  $L$  and  $T$ . However, if the units with  $c = 1$  are used, then both  $\omega_0$  and  $M_0$  may be expressed in units of  $1/T$ . Because of this relationship between the invariant frequency  $\omega_0$  and the rest mass  $M_0$  it is possible to remove the fundamental definition of relativistic mass and replace it with a derived unit of mass as it was already done in Galilean relativity (see the previous subsection). We note that the Planck constant did not enter in this relationship. The dynamical equation of point particles in the Minkowski metric may be expressed in terms of wave mass units.

The non-relativistic limits of Eqs (2) and (4), given in some textbooks [29,30], lead to additional connections between the mass-like and mass parameters of the two metrics. Thus with units  $c = 1$ , we obtain  $m = M_0$  and  $M = \omega_0$  when  $m$  and  $M$  are measured in units of  $1/T$  instead of kilogram units. Combining all relationships between the invariant parameters, an elementary particle in a Minkowski metric may be described under appropriate conditions by any one of four dynamical equations with all invariant mass-like parameters being the same:

$$m = M = \omega_0 = M_0 . \quad (6)$$

In this process the familiar concepts of mass, length and time, which are considered fundamental units of Nature, have been replaced by one fundamental unit for time, and mass and length units have been reduced to derived units  $1/T$  for mass and  $T$  for length. Thus there is no need for a circular definition of mass and the units of space and time are properly connected in

the Minkowski metric. The unit of mass was eliminated by the connection to the wave equations and the unit of space was eliminated by the Minkowski metric. The wave equations appear to have eliminated the circular definition of mass criticized by Jammer [2,17].

### 4.3 Various representations of Newton's inertial mass

According to the above results, Newton's inertial mass may be represented by different mass-like and mass parameters that arise in the fundamental (Galilean or Poincaré invariant) equations of waves and point particles. To obtain this important result, we assumed that the most basic elements of our approach were the metrics, which we used to define elementary particles, derive the invariant dynamical equations, and determine the corresponding mass-like and mass parameters.

By studying the relationships between Newton's inertial mass and these parameters, we established that all inertial observers must agree upon the value of the mass in order to identify the same elementary particle. Dynamical behavior of free elementary particles is governed by the mass-like and mass parameters and by the way they enter each invariant dynamical equation. Their presence in the Galilean and Poincaré invariant dynamical equations leads to properties that we identify physically with Newton's original concept of inertia.

In Newtonian mechanics, the property identified as inertia is commonly known as a resistance to a change in velocity of a particle with mass  $m$ , which is called the inertial mass. A generalization of this property, valid for all four fundamental theories considered above, is that the mass-like and mass parameters reflect the resistance of a particle to a change in its free particle state. The principal effect of a larger mass-like (or mass) parameter is to make it more difficult to increase the energy-like measures of the state of the system as the momentum-like parameters are increased upon application of a given force. This concept has been used to provide a working definition of a classical elementary particle [28].

## 5 The nature of mass

Mass occurs naturally in our invariant dynamical equations as a result of type of metric, definition of physical law, definition of an elementary particle, assumption of analyticity, and resulting differential equations. The central idea is that mass labels the irreps of the group of the metric, and that it also characterizes the nature of the state function during its transformation from one isometric frame of reference to another. Thus, in our approach, mass is a natural consequence of the Galilean and Minkowski metrics.

Some understanding of the inertial properties of mass can be gained from the work of Barut [31,32], who demonstrated that it is possible to take wave equations for massless particles and by separating variables find a localized solution corresponding to a rest frame frequency  $\omega_0$ . The equations then appear to have properties of a wave equation with mass proportional to the invariant frequency  $\omega_0$ . Based on the results presented in this paper, as well as on Barut's results, we conclude (Barut did not state so) that localization is the process by which inertial mass appears. What causes the localization with observed elementary particle frequencies is not fully understood for all particles, but interesting accounts of most the neutron and proton inertial masses have been given by Wilczek [33] in his "What Matters for Matter" discussions presented in *Physics Today*.

We have accomplished the task suggested by Jammer (see Sec. 1) by defining the mass of an elementary particle on its own without any specific reference to the unit of mass 'kilogram'. This elimination of the dimension of mass has allowed us to formulate the fundamental quantum theories based on the Schrödinger and Klein-Gordon equations without making any reference to the Planck constant. We have also contributed to the challenging problem of the nature of mass by showing that the mass-like and mass parameters are related to the concept of inertial mass originally introduced by Newton, and that among these parameters the invariant frequency  $\omega_0$  is the most fundamental one as the other parameters may be derived from it. Why only selected values of this parameter occur in Nature must be determined from considerations other than the free particle dynamical equations considered here. Why  $\omega_0$  takes the special values observed in Nature is not fully understood for elementary particles, but thought to arise from some underlying fields, like for instance the Higgs field [20,21].

## 6 Conclusion

We have discussed the fundamental dynamical equations for waves and point particles in space-time with both the Galilean and Minkowski metrics. The obtained equations are either Galilean or Poincaré invariant and they describe free spin-zero elementary particles that are represented by scalar wave functions, and free classical particles that are treated here as point particles. There are four invariant mass-like and mass parameters in these equations, and we have shown that these parameters are various representations of Newton's inertial mass. Our discussion of the relationships between the parameters and their physical meaning sheds a new light on the fundamental problem of the nature of inertial mass.

From the perspective of this paper, inertial mass is just a parameter that all inertial observers must agree upon to identify elementary particles. The

particular way in which the inertial mass-like and mass parameters enter each invariant dynamical equation governs its dynamical behavior, leading to properties that we identify physically with the concept of inertia. Inertial mass is a frame of reference independent description of the particle, while energy-like and momentum-like labels on the free particle are frame of reference dependent. The latter two quantities are nonetheless very useful in the description of the state of a particle relative to a given frame of reference.

It is our hope that our presentation of the concept of mass given in this paper will be helpful to scientists working in different fields of natural sciences and that it will especially benefit beginning students, who are likely to encounter conceptual difficulties with mass in their introductory physics courses. The main message of this paper is that the concept of mass occurs in Nature naturally once the metric of space-time in which we live is determined. Moreover, the mass of an elementary particle can be defined on its own, without any reference to the specific unit of mass 'kilogram'. This has important physical consequences as it allows formulating the fundamental quantum theories without formally introducing the Planck constant but using what is called here a mass-like parameter. Hence, the theories of physics may be formulated by using either the classical concept of mass with its unit of 'kilogram' and the Planck constant, or by using only one the mass-like parameter. It must also be pointed out that the theories based on the mass-like parameter can attain higher accuracy of performing computations [24,25].

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I. INTRODUCTION Inertial mass is a mass parameter giving the inertial resistance to acceleration of the body when responding to all types of force, the second law of motion provides a definition of inertial mass as the property of matter that resists changes in motion or, equivalently, as an object's inertia. Inertial mass is the name given to rotational inertia, the rotational analog of mass for linear motion. It appears in the relationships for the dynamics of rotational motion. The moment of inertia must be specified with respect to a chosen axis of rotation. Inertia about its center of mass is shown in the Figure 3. Figure 3. C. Point mass The moment of inertia of a point mass with respect to an axis is defined as the product of the mass times the distance from the axis squared. The equivalence of inertial mass and passive gravitational mass (what Eötvös is talking about) has been experimentally verified to better than one part in 10<sup>13</sup>. Proposed satellite experiments will extend that to one part in 10<sup>18</sup>. Eötvös said "I talked about classical mass. Each theory has got its own definitions and postulates. Classical physics postulated position, time, and interaction between objects. It also defined velocity and acceleration and Newton set up his laws. Its actual orbit was therefore explained by the balance between the planet's inherent centrifugal tendency (its tendency to follow the tangent to the orbit) and the pressure of the surrounding medium." First, it shows that Newton recognized properties that were commonly regarded as distinct (e.g. in the Leibnizian distinction between passive and active) as merely frame-dependent representations of the same fundamental property. All are essential to the explication of inertial mass as a measurable theoretical quantity. On the contrary, it was Newton's way of drawing attention to the precise role of inertial mass as an invariant quantity in physical interactions, underlying the various ways in which its manifestations had been previously conceived.