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# A Nonlinear Schrödinger-like Equation in the Statistical Theory of Spheroidal Bodies

Alexander M. Krot[1]

[1] Laboratory of Self-Organization Systems Modeling, United Institute of Informatics Problems of National Academy of Sciences of Belarus, Minsk, Belarus
(E-mail: alxkrot@newman.bas-net.by)

**Abstract:** A statistical theory is proposed for initial gravitational interactions of particles inside the forming cosmological bodies (molecular clouds), which have fuzzy contours and are represented by spheroidal forms. The equation for quasi-equilibrium gravitational compression of a spheroidal body in a vicinity of its mechanical equilibrium is considered initially. According to the proposed model of quasi-equilibrium gravitational compression an antidiffusion mass flow arises inside a slowly compressible gravitating spheroidal body. In this connection, the notions of antidiffusion mass flow density as well as antidiffusion particle velocity in a spheroidal body are introduced. The equations for calculating the partial derivative of the antidiffusion velocity (in the cases of absence or presence of an ordinary hydrodynamic velocity) as well as the complete derivative of the common (hydrodynamic plus antidiffusion) velocity with respect to time are obtained. As shown in this work, these equations are more general than the analogous equations derived in Nelson' stochastic mechanics. They are used for the derivation of nonlinear time-dependent Schrödinger-like equation describing a gravitational formation of a cosmological body.
**Keywords:** Molecular clouds, Initial gravitational interactions, Spheroidal bodies, Quasi-equilibrium gravitational compression, Antidiffusion mass flow, Antidiffusion velocity, Nonlinear Schrödinger-like equation.

## 1 Introduction

A statistical theory of slowly compressible gravitating cosmological body formed by a numerous of interacted particles isolated from an influence of external fields and bodies has been proposed in the works [1–6]. Within framework of this theory, the forming cosmological bodies are shown to have fuzzy contours and are represented by spheroidal forms (unlike ordinary macroscopic bodies having distinct contours). In this connection a new notion of theoretical mechanics called a *spheroidal body* has been introduced in the works [1–6] (in addition to the well-known notion as mass point). A mass point $m_0$ does not possess any geometrical sizes, on the contrary, a spheroidal body with mass $M$ has infinitely long sizes (in the physical sense of infinity, of course). By analogy with the well-known dilemma "particle–wave" (solved by means of corpuscular–wave dualism principle in the case of quantum mechanical particles) it is appropriate to consider a new concept "mass point–spheroidal

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body". Let us note that a cosmological body can be considered as a mass point at long distance of its observation or as a spheroidal body at short distance respectively.

In such spheroidal bodies, under the condition of critical values of mass density (or parameter of gravitational compression $\alpha$ [5, 6]) the centrally symmetric gravitational field arises. The tension, force, potential and energy of the gravitating spheroidal body have been determined to be of probability character [1, 2]. It has been pointed out that a spheroidal body has a clearly outlined form if the potential energy of gravitational interaction of its particles is sufficiently great and the body's mass itself is relatively small. Obviously, the spheroidal body (like the well-known objects in the theoretic physics as a single mass point, an absolutely rigid body etc.) is an *idealized* notion.

A process of slow-flowing-in time initial gravitational contraction of a spheroidal body has been investigated [3, 7]. Within framework of model of this process, the equations have been derived for description of a slow-flowing quasi-equilibrium gravitational compression of a spheroidal body in a vicinity of unstable mechanical equilibrium (initial and quasi-equilibrium) state [3, 7].

This paper considers the slow-flowing process of an initial gravitational condensation of a spheroidal body leading to origin of its gravitational field. The process of initial quasi-equilibrium gravitational compression of a spheroidal body in space within framework of the proposed "vibrating strainer" model can be interpreted on the basis of Wiener process in a space-frequency domain [8–10].

Recently L. Nottale [11, 12] has developed a new theory of the scale relativity. In Nottale's theory, both direct and reverse Wiener processes are considered in parallel; that leads to the introduction of a twin Wiener (backward and forward) process as a single complex process [11, 12]. For the first time backward and forward derivatives for the Wiener process were introduced within framework of statistical mechanics of Nelson [13, 14]. Both Nelson's statistical mechanics and Nottale's scale relativistic theory investigate families of virtual trajectories which being continuous but nondifferentiable. The important point in Nelson's works [13, 14] is that a diffusion process can be described in terms of a Schrödinger-type equation, with help of the hypothesis that any particle in the empty space, under the influence of any interaction field, is also subject to a universal Brownian motion (i.e. from the mathematical view-point, a Markov–Wiener process) [15] based on the quantum nature of space-time in quantum gravity theories or on quantum fluctuations on cosmic scale [16–18].

In the previous works [1–6], it was supposed that a weakly gravitating spheroidal body is isolated from influence of other fields and bodies, it is homogeneous in its chemical structure and has the temperature close to the absolute zero. In this paper we accept the same methodology supposing that the following assumptions are used:

1. The spheroidal body under consideration is homogeneous in its chemical structure, i.e. it consists of $N$ identical particles with the mass $m_0$.

2. The spheroidal body is not subjected to an influence of external fields and bodies.

3. The spheroidal body is isothermal and has temperature $T$ close to the absolute zero, besides $T_{\deg} < T$, where $T_{\deg} = (h^2 / m_0 k_B) n^{2/3}$ is a degeneration temperature [19], $h$ is the Planck's constant, $k_B$ is the Boltzmann's constant, $n$ is a concentration of particles.

4. The spheroidal body is weakly gravitating, i.e. it occurs in a state close to a state of instable mechanical equilibrium (when a hydrodynamic mass flow is absent though a weak mass flow takes place [20, 21]), therefore the process of gravitational contraction (compression) appears slowly developing in time (the case of *unobservable* velocities of particles composing the spheroidal body [4, 22]).

In compliance with these requirements, an attempt to derive a nonlinear equation describing a gravitational formation of a cosmological body based on model of self-organizing processes into a spheroidal body is made in this paper.

## 2 The density of antidiffusion mass flow and antidiffusion velocity into a slow-flowing gravitational compressible spheroidal body

As shown in the papers [3, 6], the dynamics of a slowly evolving process of initial gravitational condensation of a spheroidal body from an infinitely distributed substance is described by the *antidiffusion equation*:

$$\frac{\partial \rho}{\partial t} = -\mathrm{G}(t)\nabla^2 \rho, \tag{1}$$

where $\rho$ is a mass density of the spheroidal body and

$$\mathrm{G}(t) = \frac{1}{2\alpha^2} \cdot \frac{d\alpha}{dt} \tag{2a}$$

is a *gravitational compression function*, generally speaking (or a gravitational compression coefficient in some particular cases [3, 7]), $\alpha$ is a *parameter of gravitational compression* (slowly changing in time $t$), besides $\alpha > 0$ [1–7]. The solution of Eq. (1) gives us the mass density function of a *non-rotating* spheroidal body:

$$\rho(r,t) = \rho_0(t) \mathrm{e}^{-\frac{\alpha(t)}{2} r^2}, \tag{2b}$$

where $\rho_0(t) = M \left( \frac{\alpha(t)}{2\pi} \right)^{3/2}$, $M$ is a mass of the spheroidal body, $r$ is a radial coordinate.

We are going to use the general equation (1) of the slow-flowing gravitational compression; for this we shall rewrite it taking into account that gravitational compression function $\mathrm{G}(t)$ does not depend on the space variable *r*, therefore:

$$\frac{\partial\rho}{\partial t} = -\nabla\big(\mathrm{G}(t)\nabla\rho\big) = -\mathrm{div}\big(\mathrm{G}(t)\,\mathrm{grad}\,\rho\big), \tag{3a}$$

whence

$$\frac{\partial\rho}{\partial t} + \mathrm{div}\big(\mathrm{G}(t)\,\mathrm{grad}\,\rho\big) = 0\,. \tag{3b}$$

The relation (3b) reminds completely the continuity equation expressing the law of conservation of mass in a nonrelativistic system [23]:

$$\frac{\partial\rho}{\partial t} + \mathrm{div}\,\vec{j} = 0\,, \tag{4a}$$

where $\vec{j}$ is a continuum flow density. In this connection, the value in round brackets of Eq. (3b) has the sense of a mass flow density (like a conductive flow) $\vec{j}$ arising at the slow-flowing gravitational compression of spheroidal body [3, 4, 7]:

$$\vec{j} = \mathrm{G}(t)\,\mathrm{grad}\rho\,. \tag{4b}$$

For the first time, conductive (owing to diffusion or thermal conductivity) flows in dissipative systems were investigated by I. Prigogine in his works (see, for example, [20, 21]). As it follows from Eq. (4b) directly, there exists an *antidiffusion mass flow density* in a slowly compressible gravitating spheroidal body [3, 6]. Applying the equation of continuity (4a) to this antidiffusion flow density (4b) we obtain again the well-known linear antidiffusion equation (1). Since $\rho$ is a function of the space variable $r$, then in the spherical system of coordinates $\mathrm{grad}\,\rho = \frac{\partial\rho}{\partial r}\vec{e}_r = \frac{\partial\rho}{\partial r}\cdot\frac{\vec{r}}{r}$. Taking into account the fact that according to (2) the mass density $\rho$ is an exponentially decreasing function, then its derivative $\frac{\partial\rho}{\partial r} < 0$. Consequently, the direction of the antidiffusion flow density vector $\vec{j}$ is directly opposite to the basis vector $\vec{e}_r$, i.e. the vector $\vec{j}$ is directed to the spheroidal body center.

Like the particle momentum operator $\hat{p} = i\hbar\nabla$ in the quantum mechanics [24–26], we can introduce from Eq. (4b) a *velocity operator* in the case of *unobservable* velocities of particles composing a spheroidal body [4, 22]:

$$\hat{v} = \mathrm{G}(t)\nabla\ , \tag{5}$$

i.e. $\hat{v}$ is the operator of unobservable antidiffusion velocity. Taking into account this Eq. (5) the antidiffusion mass flow density (4b) of slow-flowing gravitational contraction of spheroidal body (with unobservable velocities of particles) can be written as follows [4, 22]:

$$\vec{j} = \hat{v}\rho\ . \tag{6}$$

According to Eq. (6) the continuity equation (4a) takes the form:

$$\frac{\partial\rho}{\partial t} + \mathrm{div}(\hat{v}\rho) = 0\ . \tag{7}$$

As it has been mentioned above, I. Prigogine, G. Nicolis, P. Glansdorff studied the so-called conductive (diffusive and thermal conductive) flows [20, 21] satisfying equations analogous to Eqs.(4a), (7). In this connection, along with the velocity operator $\hat{v}$ let us introduce a conductive velocity for the antidiffusion mass flow density or, simply say, *antidiffusion velocity*:

$$\vec{u} = \mathrm{G}(t)\frac{\nabla\rho}{\rho} = \mathrm{G}(t)\,\mathrm{grad}\ \ln\rho. \tag{8}$$

Obviously, the antidiffusion velocity $\vec{u}$ of the antidiffusion mass flow density satisfies the well-known continuity equation of the kind:

$$\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\vec{u}) = 0\ . \tag{9}$$

Using this continuity equation (9) we can calculate the partial derivative of the antidiffusion velocity (8) with respect to time:

$$\begin{aligned}
\frac{\partial\vec{u}}{\partial t} &= \{\frac{d\mathrm{G}(t)}{dt}\}\,\mathrm{grad}\ \ln\rho + \mathrm{G}(t)\,\mathrm{grad}\ \{\frac{1}{\rho}\frac{\partial\rho}{\partial t}\} = \\
&= \frac{d\mathrm{G}(t)}{dt}\{\frac{1}{\mathrm{G}(t)}\vec{u}\} + \mathrm{G}(t)\nabla\{\frac{1}{\rho}(-\mathrm{div}(\rho\vec{u}))\} = \\
&= \{d\ln\mathrm{G}(t)/dt\}\vec{u} - \mathrm{G}(t)\nabla\{\nabla\vec{u} + \vec{u}\,\frac{\nabla\rho}{\rho}\} = \\
&= -\mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\,\vec{u}) - \mathrm{grad}(\vec{u}^{\,2}) + \{d\ln\mathrm{G}(t)/dt\}\vec{u}.
\end{aligned} \tag{10}$$

An advantage of the antidiffusion velocity notion (8) versus the velocity operator notion (5) to be introduced is contained in the fact that the antidiffusion velocity of particles inside a slow-flowing gravitational compressible spheroidal body can become *observable* one if the mass density of spheroidal body is very small. Indeed, according to Eq. (8) if the mass density $\rho \to 0$ then the antidiffusion velocity $\vec{u} \to \infty$ (under condition that $\mathrm{grad}\rho$ be finitary). The condition of smallness for the mass density $\rho$ takes place in the molecular clouds of distributed gas-dust substance in space [27]. Thus, as a result of spheroidal body formation from an initial weakly condensed gas-dust cloud it might be a sharp *increase* of the antidiffusion velocity of particles into the forming spheroidal body under condition of finiteness of the mass density gradient. In this case it is reasonable to rewrite Eq. (10) based on the familiar formulas of vector analysis [23]:

$$\frac{1}{2}\mathrm{grad}\,\vec{u}^2 = (\vec{u}\cdot\nabla)\vec{u} + [\vec{u}\times\mathrm{rot}\,\vec{u}], \tag{11a}$$

$$\nabla^2\vec{u} = \mathrm{grad}(\mathrm{div}\,\vec{u}) - \mathrm{rot}(\mathrm{rot}\,\vec{u}). \tag{11b}$$

Taking into account Eq. (8) we can see that $\mathrm{rot}\,\vec{u} = 0,$ whence

$$\mathrm{grad}\,\vec{u}^2 = 2(\vec{u}\cdot\nabla)\vec{u}, \tag{12a}$$

$$\nabla^2\vec{u} = \mathrm{grad}(\mathrm{div}\,\vec{u}). \tag{12b}$$

Substituting Eqs.(12a,b) in Eq. (10) we obtain:

$$\frac{\partial\vec{u}}{\partial t} = -\mathrm{G}(t)\nabla^2\vec{u} - 2(\vec{u}\cdot\nabla)\vec{u} + \{d\ln\mathrm{G}(t)/\mathrm{dt}\}\vec{u}. \tag{13}$$

Taking into account Eq. (12a) again, the equation (13) can be written as follows:

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\mathrm{grad}(\vec{u}^2/2) - \mathrm{G}(t)\nabla^2\vec{u} + \{d\ln\mathrm{G}(t)/\mathrm{dt}\}\vec{u}. \tag{14}$$

The obtained equation (14) is similar to the Navier-Stokes' equation of motion of a viscous liquid [23] under conditions that a gas-dust substance of spheroidal body is isolated from influence of external fields and $\mathrm{G}(t) = \mathrm{G_S} = const.$

Now let us estimate the antidiffusion velocity (8) of particles into a spherically symmetric slow-flowing gravitational compressible spheroidal body taking account of its mass density function (2):

$$\vec{u}(\vec{r},t) = \mathrm{G}(t)\,\mathrm{grad}\,\{\ln\rho_0(t) - \alpha(t)\vec{r}^{\,2}/2\} = -\mathrm{G}(t)\alpha(t)\vec{r}. \tag{15}$$

We can see that the antidiffusion velocity $\vec{u}$ is expressed by the very simple relation (15) in the case of a spherically symmetric spheroidal body. Apropos, using approach proposed by W. Ebeling for the first time [28] the equation for spherical autowaves of magnitude of gravitational field strength

$\vec{a} = \frac{\partial \vec{u}}{\partial t} + (\vec{u}\nabla)\vec{u}$ of a slowly contracting spheroidal body has been derived in the work [29]. The obtained Eq. (15) reminds the formula the velocity of autowave front propagation [29] for gravitational strength magnitude in a remote zone of slowly compressible gravitating spheroidal body.

Along with the antidiffusion velocity $\vec{u}$ there exists an ordinary hydrodynamic velocity $\vec{v}$ (or a *convective* velocity in the sense of Prigogine [20]). In principle, the hydrodynamic velocity $\vec{v}$ of mass flow arises as a result of powerful gravitational contraction of a spheroidal body on the next stages of its evolution. The growing magnitude of gravitational field strength $\vec{a}$ induces the significant (i.e. observable) value of hydrodynamic velocity $\vec{v}$ of mass flows moving into spheroidal body. This means that the value of antidiffusion velocity (8) becomes much less than the value of hydrodynamic velocity, i.e.

$$|\vec{u}| << |\vec{v}| \ . \qquad (16)$$

Under this condition (16), a common (hydrodynamic and antidiffusion) mass flow density inside a spheroidal body satisfies the hydrodynamic equation of continuity [23]:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \vec{v}) = 0 \, . \qquad (17)$$

Taking into account Eq. (17) we can also calculate the partial derivative of the antidiffusion velocity (8) with respect to time in accord with the condition (16):

$$\begin{aligned} \frac{\partial \vec{u}}{\partial t} &= \{\frac{d\mathrm{G}(t)}{dt}\}\,\mathrm{grad}\,\ln\rho + \mathrm{G}(t)\,\mathrm{grad}\,\{\frac{1}{\rho}\frac{\partial \rho}{\partial t}\} = \\ &= \frac{d\mathrm{G}(t)}{dt}\{\frac{1}{\mathrm{G}(t)}\cdot\vec{u}\} + \mathrm{G}(t)\nabla\{\frac{1}{\rho}(-\mathrm{div}(\rho\vec{v}))\} = \\ &= \{d\ln\mathrm{G}(t)/dt\}\vec{u} - \mathrm{G}(t)\nabla\{\nabla\vec{v} + \vec{v}\,\frac{\nabla\rho}{\rho}\} = \\ &= -\mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\vec{v}) - \mathrm{grad}(\vec{v}\vec{u}) + \{d\ln\mathrm{G}(t)/dt\}\vec{u}. \end{aligned} \qquad (18)$$

As known from a fluid-like description [23], the complete time-derivative of the common (hydrodynamic plus antidiffusion) velocity $\vec{v} + \vec{u}$ inside a spheroidal body defines the common acceleration (or gravitational field strength of spheroidal body) including the partial time-derivatives and convective derivatives:

$$\vec{a} = \frac{d(\vec{v} + \vec{u})}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} + \frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}. \qquad (19a)$$

Taking into account Eq. (14) as well as Eq. (12a), the complete acceleration (19a) can be represented in the form:

$$\vec{a} = \frac{d(\vec{v}+\vec{u})}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} - (\vec{u}\cdot\nabla)\vec{u} - \mathrm{G}(t)\nabla^2\vec{u} + \{d\ln \mathrm{G}(t)/dt\}\vec{u}. \quad (19b)$$

Let us note since the mass density of spheroidal body is directly proportional to the probability volume density function according to the relation [1–6]:

$$\rho = M\Phi,$$

where $\Phi$ is a probability volume density function to locate a particle into spheroidal body, $M$ is a mass of spheroidal body, then antidiffusion velocity (8) (or (15)) can be defined by the probability volume density function:

$$\vec{u} = \mathrm{G}(t)\frac{\nabla\Phi}{\Phi} = \mathrm{G}(t)\,\mathrm{grad}\ln\Phi. \quad (20)$$

Obviously, the antidiffusion velocity (20) of probability volume flow density also satisfies Eqs.(10), (14), (15), (18) and (19a,b).

## 3 A nonlinear Schrödinger-like equation in the statistical theory of spheroidal bodies

Considerations in the works [1–6] point to an *initial quasi-equilibrium gravitational compression* occurring in a forming spheroidal body. Within framework of the proposed "vibrating strainer" model [8–10], interactions of oscillating particles inside a spheroidal body lead to the coherent displacement of particles and, as a consequence, to a resonance increase of the parameter of gravitational compression $\alpha(t)$. This means *that nonlinear phenomena* arise owing to self-organization processes [21] into a spheroidal body under its formation. These nonlinear phenomena induce nonlinear autowaves satisfying a *nonlinear* Schrödinger-like equation.

Now let us note besides the well-known linear undulatory Schrödinger equation there are its generalizations in the Nelson's statistical mechanics and the Nottale's scale relativity [11–14]. Moreover, it is not difficult to see [10] that both these equations of Schrödinger can be derived in the special case of a constant $\mathrm{G}(t)$ in the antidiffusion equation (1) when $\mathrm{G}(t) = -\hbar/2m_0$ and $\mathrm{G}(t) = -\gamma M/2\upsilon$ respectively. Obviously, this paper considers the general case of $\mathrm{G}(t)$ according to Eq. (2a), which is different from the Nelson's and Nottale's considerations. In this connection let us calculate the partial derivatives (relative to $t$) of antidiffusion velocity and ordinary hydrodynamic velocity with the aim to obtain a *nonlinear* Schrödinger-like equation by analogy with the Nelson's and Nottale's theories.

So, now let us consider again Eqs.(18), (19b) derived within framework of the statistical theory of gravitating spheroidal bodies. Taking into account the simple formulas (12a), (12b), (20), these Eqs. (18), (19b) can be rewritten in the form:

$$\frac{\partial \vec{u}}{\partial t} = -\mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\vec{v}) - \mathrm{grad}(\vec{v}\vec{u}) + \{d\ln \mathrm{G}(t)/dt\}\vec{u}; \qquad (21a)$$

$$\frac{\partial \vec{v}}{\partial t} = \vec{a} - (\vec{v}\cdot\nabla)\vec{v} + \mathrm{grad}(\vec{u}^2/2) + \mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\vec{u}) - \{d\ln \mathrm{G}(t)/dt\}\vec{u}. \qquad (21b)$$

Let us investigate some special solution of Eqs. (21a, b) in the case that the acceleration (or gravitational field strength) comes from a gravitational field potential of spheroidal body, i.e.

$$\vec{a} = -\mathrm{grad}\,\varphi_g\,, \qquad (22a)$$

under the assumption that the hydrodynamic velocity $\vec{v}$ is a gradient of a statistical action $\Im$:

$$\vec{v} = 2\mathrm{G}(t)\,\mathrm{grad}\,\Im\,. \qquad (22b)$$

Indeed, Eq. (18) points to a possible justification of Eq. (22b); in the special case of a constant $\mathrm{G}(t)$ as $\hbar/2m_0$ Eq. (22b) becomes the Nelson's formula [13]: $\vec{v} = \frac{\hbar}{m_0}\mathrm{grad}\,\Im\,.$

In this connection, $\mathrm{rot}\,\vec{v} = 0$, i.e. $(\vec{v}\cdot\nabla)\vec{v} = \mathrm{grad}(\vec{v}^2/2)$. Since $\vec{u}$ is also a gradient due to Eq. (20) as well as $\vec{a}$ and $\vec{v}$ according to Eqs. (22a,b), so that Eqs. (21a, b) become the following:

$$\mathrm{grad}\frac{\partial(\mathrm{G}(t)\,\ln\Phi)}{\partial t} = -\mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\vec{v}) - \mathrm{grad}(\vec{v}\vec{u}) + \qquad (23a)$$
$$+\{d\ln\mathrm{G}(t)/dt\}\mathrm{G}(t)\,\mathrm{grad}\,\ln\Phi;$$

$$\mathrm{grad}\frac{\partial(2\mathrm{G}(t)\Im)}{\partial t} = -\mathrm{grad}\varphi_g - \mathrm{grad}(\vec{v}^2/2) + \mathrm{grad}(\vec{u}^2/2) + \qquad (23b)$$
$$+\mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\vec{u}) - \{d\ln\mathrm{G}(t)/dt\}\mathrm{G}(t)\,\mathrm{grad}\ln\Phi.$$

Integrating these Eqs.(23a,b) and taking into account a simplification $\{d\ln\mathrm{G}(t)/dt\}\cdot\mathrm{G}(t) = d\,\mathrm{G}(t)/dt$, we can find that

$$\frac{\partial(\mathrm{G}(t)\ln\varPhi)}{\partial t} = -\mathrm{G}(t)\mathrm{div}\vec{v} - \vec{v}\vec{u} + \{d\,\mathrm{G}(t)/dt\}\cdot\ln\varPhi; \qquad (24a)$$

$$\frac{\partial(2\mathrm{G}(t)\Im)}{\partial t} = -\varphi_g - \vec{v}^2/2 + \vec{u}^2/2 + \mathrm{G}(t)\mathrm{div}\vec{u} - \{d\,\mathrm{G}(t)/dt\}\cdot\ln\Phi. \quad (24b)$$

Let us carry out a change of dependent variable:

$$\Re = \frac{1}{2}\ln\Phi; \quad (25a)$$

$$\Psi = \mathrm{e}^{\Re + i\Im}, \quad (25b)$$

where $\Im$ is defined by Eq. (22b), $i = \sqrt{-1}$. Obviously, as it follows from Eqs. (25a,b) directly

$$\Psi = \sqrt{\Phi}\cdot \mathrm{e}^{i\Im}, \quad (26)$$

so that $\Phi = \Psi\Psi^* = |\Psi|^2$ as usually. According to the first change (25a) it is not difficult to see that

$$\frac{\partial(2\mathrm{G}(t)\,\Re)}{\partial t} = -2\mathrm{G}^2(t)\nabla^2\Im - 4\mathrm{G}^2(t)\nabla\Re\cdot\nabla\Im + 2\{d\,\mathrm{G}(t)/dt\}\cdot\Re; \quad (27a)$$

$$\frac{\partial(2\mathrm{G}(t)\Im)}{\partial t} = -\varphi_g + 2\mathrm{G}^2(t)(\nabla\Re)^2 - 2\mathrm{G}^2(t)(\nabla\Im)^2 + 2\mathrm{G}^2(t)\nabla^2\Re - 2\{d\,\mathrm{G}(t)/dt\}\cdot\Re. \quad (27b)$$

Let us rewrite these two Eqs. (27a, b) as one. To this end, after multiplication of the second Eq. (27b) on imaginary unit and then addition both of Eqs. (27a, b), we can obtain the following:

$$\frac{\partial}{\partial t}\left[2\mathrm{G}(t)(\Re + i\Im)\right] = -i\varphi_g + i2\mathrm{G}^2(t)(\nabla^2\Re + i\nabla^2\Im) + i\left[\sqrt{2}\mathrm{G}(t)\nabla(\Re + i\Im)\right]^2 + 2(1-i)\{d\,\mathrm{G}(t)/dt\}\cdot\Re. \quad (28)$$

Taking into account the second change (25b) we can see that

$$\Re + i\Im = \ln\Psi; \quad 2\Re = \ln\Psi + \ln\Psi^* = \ln|\Psi|^2;$$

$\nabla(\Re + i\Im) = \nabla\ln\Psi = \nabla\Psi/\Psi$; $\nabla^2(\Re + i\Im) = \nabla^2\Psi/\Psi - (\nabla\Psi)^2/\Psi^2$, so that Eq. (28) takes the form:

$$\frac{\partial}{\partial t}\left[2\mathrm{G}(t)\ln\Psi\right] = -i\varphi_g + i2\mathrm{G}^2(t)\cdot\frac{\nabla^2\Psi}{\Psi} + (1-i)\{d\,\mathrm{G}(t)/dt\}\cdot\ln|\Psi|^2. \quad (29)$$

After some transformations and simplifications Eq. (29) can be represented as follows:

$$i2\mathrm{G}(t)\frac{\partial\Psi}{\partial t} = \varphi_g\Psi - 2\mathrm{G}^2(t)\cdot\nabla^2\Psi + \\ + i(1-i)2\{d\,\mathrm{G}(t)/dt\}\cdot\Psi\ln|\Psi| - i2\{d\,\mathrm{G}(t)/dt\}\cdot\Psi\ln\Psi\,, \tag{30}$$

whence we can obtain a nonlinear time-dependent Schrödinger-like equation of the kind:

$$i2\mathrm{G}(t)\frac{\partial\Psi}{\partial t} = [-2\mathrm{G}^2(t)\cdot\nabla^2 + \varphi_g]\Psi + 2\frac{d\,\mathrm{G}(t)}{dt}[\ln|\Psi| - i\ln\frac{\Psi}{|\Psi|}]\Psi\,. \tag{31}$$

Let us note that $\mathrm{G}(t) = \mathrm{G_S} = const$ in the *virial* (relative mechanical) *equilibrium* states of spheroidal body [3, 6], so the nonlinear time-dependent Schrödinger equation (31) becomes linear one in these special cases: for example, the time-dependent Schrödinger equation is a particular case of Eq. (31) if $\mathrm{G}(t)$ satisfies the Nelson's basic assumption [13] as well as the generalized time-dependent Schrödinger equation in the form of Nottale is a special case of Eq. (31). A specific particular case of Eq. (31) also corresponds to the Nottale's generalized time-dependent Schrödinger equation with a slowly varying diffusion coefficient $\delta D$ depending on time. So, the Nelson's and Nottale's considerations are appropriate mainly in the case of gravitational interaction of particles in a spheroidal body being in a virial equilibrium state.

Thus, the derived nonlinear time-dependent Schrödinger equation (31) describes not only the mentioned states of virial mechanical equilibrium $(\mathrm{G}(t) = \mathrm{G_S} = const)$ or quasi-equilibrium gravitational compression state close to mechanical equilibrium with a slowly varying antidiffusion coefficient $(\mathrm{G}(t) = G/t^2)$ but *gravitational instability states* with the considered resonance increase of gravitational compression of spheroidal body leading to formation of a cosmological body.

## 4 Conclusions

The main contribution of this paper is to show that gravitational compression of a spheroidal body is described by *nonlinear* Schrödinger-like equation as well as to obtain this nonlinear Schrödinger-like equation.

In Section 2, the equation (1) for the gravitational compression of a spheroidal body is considered initially. This Section investigates the density of antidiffusion mass flow into a slow-flowing gravitational compressible spheroidal body. Here the notion of *antidiffusion velocity* (8) inside a slowly compressed spheroidal body is introduced. The equations (10), (18) for calculating the partial derivative of the antidiffusion velocity with respect to time (in the cases of absence or presence of the ordinary hydrodynamic velocity) are obtained. The equation (19b) relative to the complete time-derivative of the common (hydrodynamic plus antidiffusion) velocity is also derived.

In this paper, interconnections of the proposed statistical theory of spheroidal bodies with Nelson's statistical mechanics and Nottale's scale relativistic theory are investigated. Really, both Nelson's statistical mechanics and Nottale's scale relativistic theory introduce so-called mean forward and mean backward derivatives [11-14]. It is remarkable that, in the proposed statistical theory of spheroidal bodies, the main equations (relative to *antidiffusion velocity*) have been obtained *without introducing* any *mean forward* nor *mean backward derivatives* of stochastic processes. In this regard, the proposed statistical theory differs profoundly from Nelson' stochastic mechanics [13, 14] as well as from Nottale's scale relativistic theory [11, 12, 16, 30, 31].

Moreover, the obtained main Eqs.(18), (19b) are *more general* than analogous equations in Nelson' stochastic mechanics. Indeed, within framework of the proposed statistical theory of spheroidal bodies the generalized Schrödinger equations can also be derived as in Nottale's scale relativistic theory (in the case of a constant $\mathrm{G}(t)$ the derived nonlinear Schrödinger-like equation (31) becomes the generalized Schrödinger equation). So, the Nelson's and Nottale's considerations are appropriate mainly in the case of gravitational interaction of particles in a spheroidal body being in a *virial* equilibrium state.

As noted in Section 3, this paper investigates more general dynamical states of gravitating spheroidal body. Really, the derived nonlinear time-dependent Schrödinger-like equation (31) describes not only the mentioned states of virial mechanical equilibrium $(\mathrm{G}(t) = \mathrm{G}_S = const)$ or quasi-equilibrium gravitational compression state close to mechanical equilibrium with a slowly varying antidiffusion coefficient $(\mathrm{G}(t) = G / t^2)$, but *gravitational instability states* with a resonance increase of gravitational compression of spheroidal body leading to formation of a cosmological body.

Thus, the *linear* Schrödinger equation as well as its generalizations are mentioned in this work in connection with the Nelson's statistical mechanics and the Nottale's scale relativity only. Moreover, both these equations of Schrödinger have been derived in the special case of a constant of gravitational compression function $\mathrm{G}(t)$ in the proposed antidiffusion equation when $\mathrm{G}(t) = -\hbar / 2m_0$ and $\mathrm{G}(t) = -\gamma M / 2\upsilon$ respectively. In this connection the derived equations for calculating the partial derivatives (relative to $t$) of antidiffusion velocity and ordinary hydrodynamic velocity are used to obtain a *nonlinear* Schrödinger-like equation by analogy with Nelson's and Nottale's theories. Indeed, nonlinear phenomena arise owing to self-organization processes into a spheroidal body under its formation. These nonlinear phenomena lead to nonlinear autowaves satisfying a nonlinear Schrödinger-like equation.

As mentioned above, the obtained result (relative to the nonlinear time-dependent Schrödinger-like equation (31)) has been suggested in accordance with similar conclusions of El Naschie [15] and Ord [32] that the Schrödinger equation could be universal, i.e. that it may have a large domain of applications, but with interpretations different from that of standard quantum mechanics. This main conclusion formulated with point of view of these modern quantum gravity theories confirms entirely correctness of the considered approach based on the statistical theory of spheroidal bodies.

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In intro quantum mechanics, the Schrodinger equation is a separable differential equation $\underline{when}$ the potential is independent of time. All (or most) of the examples in your textbook have potentials that are independent of time, and they are all probably solved by the method of separation of variables, thus they all have solutions called "stationary states." They are called stationary because they do not depend on time, rather they depend on position, and they are eigenstates of the Hamiltonian (they satisfy the eigenvalue equation for $H$). What does this mean? Stationary The nonlinear SchrÃ¶dinger equation is integrable when the particles move in one dimension of space. In the limit of infinite strength repulsion, the nonlinear SchrÃ¶dinger equation bosons are equivalent to one dimensional free fermions. Contents. 1 The equation.Â  The corresponding linear system of equations is known as the Zakharovâ€“Shabat system: where. The nonlinear SchrÃ¶dinger equation arises as compatibility condition of the Zakharovâ€“Shabat system: By setting or the nonlinear SchrÃ¶dinger equation with attractive or repulsive interaction is obtained. An alternative approach uses the Zakharovâ€“Shabat system directly and employs the following Darboux transformation Nonlinear schrÃ¶dinger equation in the bopp-podolsky electrodynamics: Solutions in the electrostatic case. Pietro Dâ€™avenia and gaetano siciliano. Abstract. We study the following nonlinear SchrÃ¶dinger-Bopp-Podolsky system. âˆ’âˆ†u + Ï‰u + q2Ï†u = |u|pâˆ’2u âˆ’âˆ†Ï† + a2âˆ†2Ï† = 4Ï€u2. in R3. with a, Ï‰ > 0. We prove existence and nonexistence results depending on the parameters q, p. Moreover we also show that, in the radial case, the solutions we nd tend to solutions of the classical SchrÃ¶dinger-Poisson system as a â†’ 0. Contents. 1. Introduction Notations 2. Deduction of the SchrÃ¶dinger-Bopp-Podolsky system Quasiperiodic Fourier series are important in the study of ocean waves because they provide a simpler theoretical interpretation and faster numerical implementation of the NLFA, with respect to the IST, particularly with regard to determination of the breather spectrum and their as.Â  Breather Turbulence: Exact Spectral and Stochastic Solutions of the Nonlinear SchrÃ¶dinger Equation. by Alfred R. Osborne. Nonlinear Waves Research Corporation, Alexandria, VA 22314, USA. Like the nonlocal NLS equation, these solutions may have singularities. However, by suitable constraints of parameters, nonsingular breather solutions are generated. Besides, by taking a long wave limit of these obtained soliton solutions, rogue wave solutions and semi-rational solutions are derived. For the two dimensional NLS equation, rogue wave solutions are line rogue waves, which arise from a constant background with a line profile and then disappear into the same background.Â  Solitons in a nonlinear SchrÃ¶dinger equation with PT-symmetric potentials and inhomogeneous nonlinearity: Stability and excitation of nonlinear modes. Phys.Rev.A 2015;92(2):368â€“374. View Article.