POISSON' "*SLOUGHS*" IN HIS FINAL WORKS IN LIFE,

"*A STUDY OF MATHEMATICAL PHYSICS.*"

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Contents

1. Preface

1.1. Remark on continuum for heat theory and fluid dynamics.

[1] Duhamel [4] comments on the continuum and Poisson's paper [25] :

> We explain afterward how he do with Mr. Poisson obtain the same equation with Navier has made known in 1821, with talking the molecular actions, and in considering the corps as continue. This method inspecting the molecular actions is originally due to Laplace, who has deduced from this a nice theory of capillary action. Mr. Navier has obtained afterward the nice idea to deduce the theory of elastic solid ; however, both of the mathematicians have supposed the molecules of adjacent corps, and Poisson is the first of coincidence with calculations with the physical structures. In addition to, although the hypotheses of continuum theory have been actually so inexact, however, have played big roles in the science, In the roles, have played, the theories by Mr. Laplace have welcomed by the researchers. This observation on the molecular activities, in the bulk of special problems, above all, in theory of the elastic bodies, it has the very countless merits to have to sweep out the all special hypotheses. Mr. Poisson emphasizes the merit of this method ; we will reproduce textually this passage from his Mémoire. [4, pp.98-99] (trans. and italics mine.)

We would like to point out Poisson' sloughs in his final works in life :

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[1]Siméon Denis Poisson : Born 21/June/1781 at Pithiviers, dead 25/April/1840 at Sceaux, Poisson enters l'Ecole Polytechnique in 1798 and there, will make career as professor. His works are numerous (almost 400 published) and amount specially to applied mathematics and to the physics. (HP of fdp : Fédération Denis Poisson).

1. He proposes the cause of rise/fall of capillary surface is due to the variation of density. Today's explanation is due to surface tension. (Paty 1.)
2. Another equation of fluid dynamics, which is the original of the Navier Stokes equations. (Part 2.)
3. He conjectures the proof on the exact differential will be defect. (Part 2.)
4. The difference between Lagrange's series and the Fourier Series. (Part 2.)
5. Another equation of heat different from Fourier. (Part 3.)

In the table 1, Poisson's second books [28, 29] and third book [30] seem to be contradict in the order of publishing year on describing title pages, (Poisson says the second book is [30]), however, he explains as follows :

> This *Theorie mathématiques de la chaleur*, (*Mathematical theory of heat*) will form the second part of *Un Traité de Physique mathématiques*, [2] (*A Study of Mathematical Physics*), where I propose to consider successively, without hesitation for any order preventing the progress, the diverse questions of the physics to which I will apply the analysis. The primary part of this *Traité* is the *Nouvelle théorie de l'Action capillaire*, the (*New Theorie of the Capillary Action*), published in 1831. [30, p.6]. [3]

Consequently, he doesn't hesitate for 'any order' preventing the progress, where, we think, he seems to intends to slough from the old-fashoned order in the wide meaning, because he struggles for the truth in rivalry relations in his life. So we use our title from this phrase. [4]

Table 1. The three books consisted of *A Study of Mathematical Physics.*

		1	2	3
	name and bibliography	New theory of Capillary Action 1831 [27]	Study of Mechanics 1833 [28, 29]	Analytical Theory of Heat 1835 [30]
1	pages+figures	326+1	[28] : 696+4, [29] : 782+3 total : 1478+7	543+1
				2347+9
2	rivalry & preceding studies	Laplace 1805 [10], Gauss 1830 [6]	Langarnge 1788 [8], Laplace 1798-05 [9]	Fourier 1822 [5]
3	newness & uniqueness	• rising by density variation • adaptation to continuum	• mathematical principles of mechanics • analysis of exact differential • Lagrange's summation & Fourier's series • uptodate astronomy	based on this general hypothesis of a molecular radiation. (cf. § 7.)
4	composition of capillary forces	• attraction • repulsion		• attraction by molecule • repulsion by calorific
5	apprication of Legendre's function	measurement of capillarity	computation of ellipsoid without Legendre's function	figure of earth

[2](⇓) There doesn't exist any book entitled this name. He published also *Traité de Mécanique*, in 1811. cf. [17], however, this is neither identical with [17] in respect to the title, nor the publishing date.

[3](⇓) cf. [27].

2. Part 1. *New Theory of the Capillary Action.*

2.1. The general conception of capillary action. [5]

Poisson discuss the attractive and repulsive forces in the hydrostatics, in the hydrodynamics, and in the heat theory, citing his paper [25]. cf. [27, p.30]. Poisson mentions p as $p = \varpi + \Delta$, he defines $f(r)$ the measure of the molecular action in the distance r and related with the unit of the volume.

> Hence, to satisfy the conditions of preceding article, the sum which p represents can't be reduced to an integral, and it must be equal with two terms ϖ and Δ of its complete value. Although the smallness of ε, the latter term can effectively become comparable and same measure than the former, when the two attractive and repulsive forces, which dues to $f(r)$, are mutually, extremely great in comparison to the difference. We show this point the development and the examples which I have given in my memoir on the general equation of the equilibrium and the motion of the solid elastic corps and fluids (cf. [25]). [27, p.30] [6]

In the other hand, Poisson cites [25] in his book on the capillary action [27], discussing the same theme of the development and example of the two attractive and repulsive forces. Although deducing into the same results of the fundamental formula respectively as follows, Poisson asserts his own discussion on the attractive and repulsive forces, whose method comes from the essential conception among the hydrodynamics, hydrostatics and heat theory. Followings coincide respectively in expression of the formulae.

Laplace [10, p.19] (R and R' are the radii of the priciple curvatures, respectively) :

$$\frac{1}{R} + \frac{1}{R'} = \frac{(1+q^2)\frac{dp}{dx} - pq\left(\frac{dp}{dy} + \frac{dq}{dx}\right) + (1+p^2)\frac{dq}{dy}}{(1+p^2+q^2)^{\frac{3}{2}}}, \quad p = \frac{dz}{dx}, \quad q = \frac{dz}{dy} \tag{1}$$

Gauss [6, p.64-65] (R and R' are the same with Laplace) :

$$\begin{aligned} \frac{d\xi}{dx} + \frac{d\eta}{dy} &= -\zeta^3\Big[\frac{d^2z}{dx^2}\Big\{1 + \Big(\frac{dz}{dy}\Big)^2\Big\} - \frac{2d^2z}{dx.dy}.\frac{dz}{dx}.\frac{dz}{dy} + \frac{d^2z}{dy^2}\Big\{1 + \Big(\frac{dz}{dx}\Big)^2\Big\}\Big] = \frac{1}{R} + \frac{1}{R'}, \\ where, \quad \zeta^3 &= \Big[1 + (\frac{dz}{dx})^2 + (\frac{dz}{dy})^2\Big]^{-\frac{3}{2}}. \end{aligned} \tag{2}$$

Poisson [27, p.61, p.99] (λ and λ' are the radii of the priciple curvatures, respectively) :

$$\frac{1}{\lambda} + \frac{1}{\lambda'} = \frac{\left[1 + \left(\frac{dz}{dy}\right)^2\right]\frac{d^2z}{dx^2} - 2\frac{dz}{dx}\frac{dz}{dy}\frac{d^2z}{dxdy} + \left[1 + \left(\frac{dz}{dx}\right)^2\right]\frac{d^2z}{dy^2}}{\left[1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2\right]^{\frac{3}{2}}} \tag{3}$$

These fundamental formulae are conventionally deduced after discussing personally the molecular activity between the attractive and repulsive forces, even if it were differed with each other. Poisson's one is based on the essential concept dues to [25].

Their form is the same with that of the equations of the *Mécanique céleste* (Celestial mechanics) ; however, the expressions in definite integrals of two special constants which they include are very different, so that their numerical values would be equally if, instead of determining it with the experience, we would be capable to calculate them directly owing to their analytic expressions, this one, which would be necessary that we would know the law of the action of the

[4]To establish a time line of these contributors, we list for easy reference the year of their birth and death: Kepler(1571-1630), Newton(1642-1727), Daniel Bernoulli(1700-82), Euler(1707-83), d'Alembert(1717-83), Lagrange(1736-1813), Laplace(1749-1827), Legendre (1752-1833), Fourier(1768-1830), Gauss(1777-1855), Poisson(1781-1840), Bessel(1784-1846), Navier(1785-1836), Cauchy(1789-1857), Dirichlet(1805-59), Stokes(1819-1903), Riemann(1826-66).

[5](⇓) The original title is : *Nouvelle Théorie de l'Action Capillaire.* [27]

[6]For example, cf. [25, pp. 98-99, p.134, pp.170-1]

tube on the liquid and of liquid on itself.

With the rules known of the calculation of the variation, we determine the surface unknown of the liquid which makes this sum a *minimum*, and as we see, we find at once, the general equation of this surface and the equation particular to its contour, this one, which is the characteristic merit of the method which Mr. Gauss has followed. But, this great prodigious mathematician having started from the similarly given physics with Laplace, and having no more consider the variation of the density at the extremity of the liquid, which he has regarded, in contrary, as incompressible in all the parts, the objections which is structured against the theory of Laplace applies equally to his, which isn't different with the other from the manner to formulate the equations of the equilibrium.

General consequence which we will make from our theory, it is here that the phenomena of the capillarity are due to the molecular action, modified, not only with the curvature of the surface, as Laplace has discussed, but also with the particular state of the liquids at their extremities.

Table 2. The three papers/a book on the capillary action

		1	2	3
	Name and bibliography	Laplace(1749-827) 1798-05 [9] 1805 [10], et.al.	Gauss(1777-855) 1830 [6]	Poisson(1781-840) 1831 [27]
1	language, pages	French, 78	Latin, 49	French, 302
2	restrictions	incompressible fluid	according to Laplace' physics, incompressible fluid	
3	composition of capillary forces	• attraction • repulsion (after 1819)	• gravity • the attractive force • for these forces, we will designate the $\prec$ *characteristic F* $\succ$ such that the inverse-directional distance is used.	• universal attraction • molecular attraction • calorific repulsion (§ 129)
4	mathematical newness	• two special constants • equation of surface using principal radii of curvature • adaptation to continuum	• introduction of variation problem from Lagrange (§ 18) • analysis from geometry (§ 20) • comparison of efficiency of methods between analysis and geometry (§ 25) • reduction from sextuplex integral to quadruplex integral (§ 16) • principle of virtual velocity	• adaptation in both theory and practice to continuum • analysis of fluidity(§ 62) • the difference between fluid and solid corps.(§ 131) • principle of heat theory (§ 129) • calculation in aid of elliptic function by Legendre • point of arête vive (§ 112 and ff.) • point of inflection (§ 54 and ff.). • reduction from multiplex integrals such as quitiplex (§ 17), sextuplex (§ 18)

2.2. **The modeling and proof of rise/fall in liquid.**

We discuss the problem of capillary action by Poisson (1831) from the history of mathematical physics, or, the modeling and calculation of the rise/fall of the liquid in the neighborhood of wall.

He supposes the mutual action of attraction between the molecules, $\rho^2\varphi(r)\omega\omega' ds ds'$ with the function $\varphi(r)$ of distance r between two molecules. He separates the domain of the liquid into four parts C, C', D and D', of which the two are near the wall : C locates over C', and other two, in the liquid, D locates over D'. He seeks the unknown Δ from the $2R' - R \equiv \Delta$, where, R

Table 3. The theory of capillary action and other fluid theories by Poisson

		1	2	3	4
	title	New theory of capillary action	Distribution of heat of solid corps	General equations of the equilibrium and of the motion of elastic corps and of fluids	Mathematical theory of heat
1	bibliography and pages	1831 [27], 302	1823 [22], 144 1823 [23], 152	1829-31 [25], 174	1835 [30], 552
2	Constitution of action	(• universal gravity) (§ 129) • attraction of molecule • caloric (§ 2, § 52, § 129 and ff.) • calorific repulsion	• attraction of molecule • calorific repulsion(3)	• attraction of molecule • calorific repulsion (1)	• attraction of molecule • calorific repulsion
3	special constants	F and H	k	K and k	k
4	Laplace' work of capillary action	rewrite			citing ellipsoid of revolution
5	Gauss' work of capillary action	calculation of variation			none
6	Legendre's elliptic function	apply			apply
7	fluidity	refer	refer	refer	refer

Table 4. The cross reference table of critics on the capillary action

	1	2	3	4
	Name	Laplace	Gauss	Fourier
1	Laplace			
2	Gauss	Gauss says : in p.5, since not only he developed clearly incorrect argument but also showed the false proofs : we consider that calculations in pages p.44 ff. *are the vain effects.* (§ 5)		
3	Poisson	• Poisson uses chapter 1 for criticizing extensively. • Totally, he cites Laplace in about 20 pages of his book. • Although the principles on which is based, the theory which makes the object of this oeuvre different essentially from those of the *Mécanique céleste*, however, we have obtained from equations of the same form with Laplace has given. (§ 128)	• Gauss uses the same physics like Laplace. • He negtects variation of density.	The results are same, however, the methods are different.

and R' the actions from the liquid and from the wall. Under the condition of constant density

Table 5. The points of argument on the capillary action

	1	2	3	4
	Name	Laplace	Gauss	Poisson
1	two special constants or two functions	K and H	(as function) φ and Φ	• Using K and H with different integral method from Laplace and get the same result. • F and H with different method from Laplace. • this one motivates Poisson to examine it.
2	variation of density	invariable	invariable	variable
3	compressiblity	imcompressible	imcompressible	• compressible or imcompressible.
4	fluidity	non refer	refer	• explanation of the differences between sloid corp and fluid. • complete fluidity.
5	viscosity			• the differences between experiment and theory due to viscosity. • viscosity is intermediate between sloid corp and fluid. • however, the concrete analysis is none until Stokes in 1859 and Leynolds in 1880.

of liquid,

$$R = \rho^2 \int\int\int\int\int\int \varphi(r)\frac{z+z'}{r}(1-ku)(1+k'u')dzdz'dudu'dsds',$$

in putting $r^2 = x^2 + (u+u')^2 + (z+z')^2$ and putting $k = k' = 1$,

$$q \equiv 2\rho^2 \int\int\int\int\int \varphi(r)\frac{z+z'}{r}dzdz'dudu'dx, \quad q' \equiv 2\rho\rho' \int\int\int\int\int \varphi'(r)\frac{z+z'}{r}dzdz'dudu'dx,$$

where, q and q' the quantities, ρ and ρ' densities of two material, dz, dz', du, du', dx, each elements of the distances. Using c the contour and $R = \int qds = cq$, and integrating the function, he calculates the quantities of action Q, Q', P defined Q in D, Q' in D' and P in C', under the condition of equilibrium $Q+Q'+P = 0$ in D, and, he gets $P = -2cq$, $Q' = R = cq$, $Q = \Delta = cq$. (On the Q, he shows another direct method.)

By his hypothesis, it turns finally $q = q'$, $\rho = \rho'$, because of the constant density, namely, it means that the materials are equal between the tube and liquid. From this contradiction, he concludes the rise/fall dues to the abrupt change of variation in density of liquid near the wall.

If we calculate Q without using the equilibrium in D, then we get as follows.

$$\begin{aligned} Q &= \rho^2 \int\int\int\int\int\int \varphi(r)\frac{z+z'}{r}dzdz'dudu'dxds, \\ r^2 &= x^2 + (u+u')^2 + (z-z')^2. \end{aligned}$$

in conserving all the notations of the number cited, and integrating in respect to $z$and z', from the plane GF to its tangential plane.

$$Z = \int_0^{y+\theta u}\int_0^{y-\theta u} \varphi(r)\frac{z'-z}{r}dzdz'.$$

from the above, we conclude

$$\int\int\int Zdudu'dx = 2\int_0^{\infty}\Phi(r')rdr\int_0^{\infty}\int_0^{\infty}\frac{dud\zeta}{(1+\zeta^2)\left[1+u^2(1+\zeta^2)\right]},$$

namely,

$$\int\int\int Zdudu'dx = \frac{\pi\theta}{\sqrt{1+\theta^2}}\int_0^\infty r\Phi'(r)dr,$$

in effectuating the integration relative to u, and next, that which responds to ζ.

Owing to this reduction of the integral in respect to z, z', u, u', x, and in putting for θ its value $-\cot\omega$, the expression of Q will turn into

$$Q = -\pi\rho^2\int_0^\infty r\Phi'(r)dr.\int\cos\omega ds. \tag{4}$$

In integrating by parts, it turns into

$$\int_0^\infty r\Phi'(r)dr = \frac{1}{2}\int_0^\infty r^3\Phi(r)dr = \frac{1}{8}\int_0^\infty r^4\varphi(r)dr\ ;$$

from the above, we conclude with

$$q = \frac{\pi\rho^2}{8}\int_0^\infty r^4\varphi(r)dr,$$

from (4), we get :

$$Q = -q\int\cos\omega ds. \qquad Q = -cq\cos\omega,$$

where, $\int ds = c$.

3. Part 2. *Study of Mechanics.*

The *material* is all this one, which can affect our sense of a certain manner. The *corps* are the portion of material limited in all sense, and which have, in consequence, a *form* and a *volume* determined. We call *mass* of a corps, the quantity of material of which it is composed.

A *material point* is a corps infinitely small in all the dimensions ; so that the length of all the line composed in it interior, is infinitely small, namely, less than all length which we can assign. We can regard a corps of finite dimensions, as a assemble of an infinity of material points and its mass as the sum of all their masses infinitely small.

3.1. **Another equation of fluid dynamics, which is the original of the Navier Stokes equations.**

$$(7\text{-}9)_{Pf}\quad \begin{cases} \rho(X-\frac{d^2x}{dt^2}) = \frac{d\varpi}{dx}+\beta(\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}),\\ \rho(Y-\frac{d^2y}{dt^2}) = \frac{d\varpi}{dy}+\beta(\frac{d^2v}{dx^2}+\frac{d^2v}{dy^2}+\frac{d^2v}{dz^2}),\\ \rho(Z-\frac{d^2z}{dt^2}) = \frac{d\varpi}{dz}+\beta(\frac{d^2w}{dx^2}+\frac{d^2w}{dy^2}+\frac{d^2w}{dz^2})\\ \text{where } \varpi = p+\frac{\alpha}{3}(K+k)(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}), \end{cases} \tag{5}$$

$$\Rightarrow^{*}\quad \begin{cases} \rho(\frac{Du}{Dt}-X)+\frac{dp}{dx}+\alpha(K+k)\Big(\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}\Big)+\frac{1}{3}\alpha(K+k)\frac{d}{dx}\Big(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\Big)=0,\\ \rho(\frac{Dv}{Dt}-Y)+\frac{dp}{dy}+\alpha(K+k)\Big(\frac{d^2v}{dx^2}+\frac{d^2v}{dy^2}+\frac{d^2v}{dz^2}\Big)+\frac{1}{3}\alpha(K+k)\frac{d}{dy}\Big(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\Big)=0,\\ \rho(\frac{Dw}{Dt}-Z)+\frac{dp}{dz}+\alpha(K+k)\Big(\frac{d^2w}{dx^2}+\frac{d^2w}{dy^2}+\frac{d^2w}{dz^2}\Big)+\frac{1}{3}\alpha(K+k)\frac{d}{dz}\Big(\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}\Big)=0, \end{cases}$$

($\Downarrow$) Here, $\alpha(K+k)$ is the constant to the tensor function with the main axis (the normal stress) of Laplacian. $\frac{1}{3}\alpha(K+k)$ corresponds to the coefficient of grad.div term. In today's NS equations, the ratio of coefficient attached to the term of the tensor function with the main axis (the normal stress) of Laplacian to that of grad div : $\frac{\text{coefficient of tensor}}{\text{coefficient of grad div}} = 3$, like Poisson deduced in $(7\text{-}9)_{Pf}$ and Stokes' $(12)_S$ through the tensor by Saint-Venant. By Prandtl [32, p.259] in 1934, we had have to wait by the time, when including this ratio of two coefficients, as what is called the NS equations were expressed in fluid equation. ($\Uparrow$) Stokes pointed out the coincidence with Poisson using the correspondence:

$\varpi = p + \frac{\alpha}{3}(K+k)\Big(\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}\Big)$ which then gives $\nabla\varpi = \nabla p + \frac{\beta}{3}\nabla(\nabla\cdot\mathbf{u})$. Stokes also commented:

> The same equations have also been obtained by Navier in the case of an incompressible fluid (Mém. de l'Académie, t. VI. p.389) [7], but his principles differ from mine still more than do Poisson's. [33, p.77, footnote]

He further stated:

> Observing that $\alpha(K+k) \equiv \beta$, this value of ϖ reduces Poisson's equation $(7\text{-}9)_{Pf}$ (=(5) in our renumbering) to the equation $(12)_S$ of this paper.

3.2. **The conjecture of defect of Proof on Exact differential.**

Poisson [26] comes to a close[8] in appending his opinion about the proof of exact differential in the last pages of [25, pp.173-4]. His conjucture is based on the preceding analysis in [23, pp.382-3].

The proof of the conservation in time and space of an exact differential was discussed by Lagrange, Cauchy, Stokes, and others. The herein-called "Poisson conjecture" in 1831, cited in the Introduction as one of our main motivations for this study, It had its beginnings with the incomplete proof by Lagrange [8]. However, thereafter, Cauchy [2] had presented a proof as early as 1815, while Power [31] and Stokes [33] had tried by other methods.

To date Cauchy's proof is still considered to be the best. Poisson concludes the proof is defect, and even the equation made of tenscendentals satisfy with exact differential at the original time of movement, the equations satisfy no more with it during all the time:

4. Part 3. *Mathematical Theory of The Heat*

In sum, Poisson's [30] is extending his railroadline of analysis of heat motion on the hypothesis based on the molecular radiation. This is the extending effort since the analysis on fluid motion [25], and hydrostatics [27]. cf. [30, p.13]. This comes from the rivalry to Navier and Fourier. Poisson ignores Navier as Arago [1] says, however, to Fourier, Poisson refutes him in the several papers since the paper [18] on the abstract of Fourier's initial work in 1808. Poisson gets to coincide the equation of interior motion of heat with Fourier's as follows, though Poisson's ($(8)_{PS4}$) deducing method is different with Fourier's ($(1)_{PS11}$). (cf. [30, p.347])

$$(8)_{PS4} \qquad c\frac{du}{dt} = k\left(\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right) + \frac{dk}{du}\left(\frac{du^2}{dx^2} + \frac{du^2}{dy^2} + \frac{du^2}{dz^2}\right). \tag{6}$$

$$(1)_{PS11} \qquad \frac{du}{dt} = a^2\left(\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right), \qquad \frac{k}{c} = a^2, \tag{7}$$

where, u is the heat, k and c are the conductibility and the specific heat of the material.

References

[1] D.F.J.Arago, *Note du Rédacteur*, Annales de chimie et de physique, **39**(1829), 107-110. (This is following with Navier[?], 99-107).

[2] A.L.Cauchy, *Mémoire sur la Théorie des Ondes, 1815*, Savants étrangers, **1**(1827), 1 partie §§3,4 et 2 partie §§4,5. (Remark : this is the same as *Mémoire sur la Théorie des Ondes à la surface d'un fluid pesant d'un profondeur indéfinie*, Œuvres de Cauchy, 1882, serie (1), t. 1, 5–318.)

[3] G. Darboux, *Œuvres de Fourier. Publiées par les soins de M.Gaston Darboux*, Tome Premier, Paris, 1888, Tome Second, Paris, 1890.

[4] J.M.C. Duhamel, (Book review) *Mémoire sur l'Équilibre et le Mouvement des Corps élastiques ; par M.Poisson*, Bulletin des sciences mathématiques, astromatiques, physiques et chimiques, **11**(1829), 98-111. (The title number : No.35.)

[7] (⇓) cf. Navier [15].

[8] This note's accepted date is signed as Lu : 2/mars/1829.

Table 6. The principles of the mechanics in *A Study of Mechanics.*

	volume	1	2	3	4	5	6
	contents	Statics 1	Dynamics 1	Statics 2	Dynamics 2	Hydro statics	Hydro dynamics
	articles no.	§1-§109	§110-§255	§256-§349	§350-§573	§574-§644	§645-§696
	pages no. of original.	1-202	203-496	497-696	1-502	503-644	645-696
	pages no. of translation.	1-122	123-280	281-389	390-653	654-737	738-797
1	infinitesimal Analysis	13, 14					
2	equilibrium	45					
3	Mechanics	47					
4	homogeneity of quantities	29, 31	134				
5	virtual velocities	45, 56		367, 387	601, 602	658–660	781
6	universal attraction		264				
7	least action		171, 174, 175		651, 652		
8	Dynamics				396		
9	reaction equal and contrary to the action				628		
10	motion of the center of gravity				629		
11	three moments of inertia				426, 427, 460		
12	conservation of the area				652	654	
13	conservation of the motion				652	692	
14	d'Alembert				390–392, 401, 402, 607		769
15	superposition of the small motions				626		761
16	vital forces				641, 644, 651, 652	701	781, 793
17	Hydrostatics					692	
18	equality of pressure in all directions					655, 656, 660	738,746

[5] J.-B.-J. Fourier, *Théorie analytique de la chaleur. Deuxiéme Édition*, Paris, 1822. (This is available by G.Darboux [3] [Tome Premier] with comments). → http://gallica.bnf.fr/ark:/12148/bpt6k1045508v (Remark. This is the original book version of BnF : *Bibliothèque Nationale France*, not the book edited by G.Darboux. After p.32, there are 4 pages of pp.23-26, however, p.33ff are followed after p.26. Total pages are 639 plus the 2 pages of figures, while the book edited by G.Darboux is consisted of 563 pages.)
bibitemGau1828 C.F.Gauss, *Disquisitiones generales circa superficies curvas*, Gottingae, 1828, *Carl Friedrich Gauss Werke VI*, Göttingen, 1867. (We can see today in : "*Carl Friedrich Gauss Werke VI*", Georg Olms Verlag, Hildesheim, New York, 1973, 219-258. Also, *Anzeigen eigner Abhandlungen, Götingische gelehrt Anzeigen*, 1927, "*Werke VI*", 341-347.) (Latin)

[6] C.F.Gauss, *Principia generalia theoriae figurae fluidorum in statu aequilibrii*, Gottingae, 1830, *Carl Friedrich Gauss Werke V*, Göttingen, 1867. (Similarly: "*Carl Friedrich Gauss Werke V*", Georg Olms Verlag, Hildesheim, New York, 1973, 29-77. Also, *Anzeigen eigner Abhandlungen, Götingische gelehrt Anzeigen*, 1829, as above in "*Werke V*", 287-293.)

[7] J.L.Lagrange, *Mémoire sur la théorie du mouvement des fluides*, Œuvres de Lagrange publiées par les soins de M.J.-A. Serret, Vol. **4** 1869, 695–748. (Lu : 22/nov/1781.) → http://gallica.bnf.fr/ark:/12148/bpt6k229223s

[8] J.L.Lagrange, *Méchanique analitique*, Paris, 1788. (Quatrième édition d'après la Troisième édition de 1833 publiè par M. Bertrand, *Joseph Louis de Lagrange, Oeuvres*, publièes par les soins de J.-A. Serret et Gaston Darboux, **11/12**, Georg Olms Verlag, Hildesheim·New York, 1973.) (J.Bertrand remarks the differences between the editions.)

[9] P.S.Laplace, *Traité de méchanique céleste*, Ruprat, Paris, 1798-1805, 1-66. (We use this original printed by Culture et Civilisation, 1967.)

[10] P.S.Laplace, *Supplément à la théorie de l'action capillaire*, Tome Quatrième, Paris, 1805, 1-78. (op. cit. [9].)

[11] P.S.Laplace, *Traité de méchanique céleste./* •§4 *On the equilibrium of fluids./* •§5 *General principles of motion of a system of bodies./* •§6 *On the laws of the motion of a system of bodies, in all the relations mathematically possible between the force and velocity./* •§7 *Of the motions of a solid body of any figure whatever./* •§8 *On the motion of fluids*, translated by N. Bowditch, Vol. I §4-8, pp. 90-95, 96-136, 137-143, 144-193, 194-238, New York, 1966.

(The inside cover of this book reads : the present work is a reprint, in four volumes, of Nathaniel Bowditch's English translation of volumes I,II,III and IV of the French-language treaties *Traité de Méchanique Céleste*, by P.S. Laplace. The translation was originally published in Boston in 1829, 1832, 1834 and 1839, under the French title, *"Méchanique Céleste"*, which has now been changed to its English-language form, *"Celestial Mechanics."*)

[12] P.S.Laplace, *On capillary attraction, Supplement to the tenth book of the Méchanique céleste*, translated by N. Bowditch, Vol. IV, pp.685-1018, New York, 1966. (op. cit. [11].)

[13] S. Masuda, *Historical development of classical physico-mathematics*, Scholars' Press, 2014.

[14] S. Masuda, *Historical development of classical physico-mathematics*, Scholars' Press, 2014.

[15] C.L.M.H.Navier, *Mémoire sur les lois du mouvement des fluides*, Mémoires de l'Academie des Sience de l'Institute de France, **6**(1827), 389-440. (Lu : 18/mar/1822.) → http://gallica.bnf.fr/ark:/12148/bpt6k3221x, 389-440.

[16] S.D. Poisson, *Sur les intégrales définies*, Nouveau Bulletin des Sciences, par la Société Philomatique, Paris. Avril. **42**(1811), 243-252. (referred : [19, p.219])

[17] S.D. Poisson, *Traité de Mécanique*, vol 1-2, Chez M^{me} veuve Courcier, Imprimeur-Libraire pour les Mathematiques, Paris. 1811. (Company of widow Courcier, for printing and publishing for the mathematics.) (vol.1) → http://gallica.bnf.fr/ark:/12148/bpt6k903370 (vol.2) → http://gallica.bnf.fr/ark:/12148/bpt6k90338b

[18] S.D. Poisson, *Mémoire sur la Propagation de la Chaleur dans les Corps Solides*, Nouveau Bulletin des Sciences par la Société philomatique de Paris, t.I, 112-116, no.6, mars 1808. Paris. (Lu : 21/déc/1807) (Remark. The author of paper is named as Fourier, for the report of Fourier's undefined version, however, the signature in the last page is 'P' meant Poisson.) → [3] vol.2, 215-221. → http://gallica.bnf.fr/ark:/12148/bpt6k33707

[19] S.D. Poisson, *Mémoire sur les intégrales définies*, (1813), J. École Polytech., Cahier **16**, **9**(1813), 215-246. → http://gallica.bnf.fr/ark:/12148/bpt6k4336720/f220

[20] S.D. Poisson, *Suite du Mémoire sur les intégrales définies, imprimé dans le volume précédent de ce Journal*, J. École Polytech., Cahier **17**, **10**(1815), 612-631. → http://gallica.bnf.fr/ark:/12148/bpt6k433673r/f614 (followed from [19].)

[21] S.D. Poisson, *Mémoire sur l'intégration de quelques équations linéaires aux différences partielles, et particulièrement de l'équation générale du mouvement des fluides élastiques*, Mémoires de l'Académie royale des Siences, **13**(1818), 121-176. (Lu : 19/juillet/1819.) (referred : [22, p.139])

[22] S.D. Poisson, *Mémoire sur la Distribution de la Chaleur dans les Corps solides*, J. École Royale Polytech., Cahier **19**, **12**(1823), 1-144. (Lu : 31/déc/1821.) → http://gallica.bnf.fr/ark:/12148/bpt6k433675h

[23] S.D. Poisson, *Second Mémoire sur la Distribution de la chaleur dans les corps solides*, J. École Royale Polytech., Cahier **19**, **12**(1823), 249-403. (Lu : 31/déc/1821.)

[24] S.D. Poisson, *Mémoire sur l'Équilibre et le Mouvement des Corps élastiques*, Mémoires de l'Académie royale des Siences, **8**(1829), 357-570. (Lu : 14/apr/1828.)

[25] S.D. Poisson, *Mémoire sur les équations générales de l'équilibre et du mouvement des corps solides élastiques et des fluides*, (1829), J. École Royale Polytech., **13**(1831), 1-174.

[26] S.D.Poisson, *Mémoire sur l'Équilibre des fluides*, Mémoires de l'Academie royale des Siences, **9**(1830), 1-88. (Lu : 24/nov/1828.) → http://gallica.bnf.fr/ark:/12148/bpt6k3224v, 1-88.

[27] S.D. Poisson, *Nouvelle théorie de l'Action capillaire*, Bachelier Pére et Fils, Paris, 1831.

[28] S.D. Poisson, *Traité de Méchanique* (1), (2), Bachelier, Imprimeur-Libraire pour les Mathématiques, Paris, 1833. (1)→

[29] S.D. Poisson, *Traité de Méchanique* (2), Bachelier, Imprimeur-Libraire, Paris, 1833. →

[30] S.D. Poisson, *Théorie mathématique de la chaleur*, Bachelier Pére et Fils, Paris, 1835. → http://www.e-rara.ch/doi/10.3931/e-rara-16666

[31] J.Power, *On the truth of the hydrodynamical theorem, that if $udx + vdy + wdz$ be a complete differential with respect to x, y, z, at any one instant, it is always so*, Cambridge Philosophical Transactions, **7**(1842), (Part 3), 455–464.

[32] L.Prandtl, *Fundamentals of hydro-and aeromechanics*, McGrawhill, 1934. (Based on lectures of L.Prandtl (1929) by O.G.Tietjens, translated to English by L.Rosenhead. 1934.)

[33] G.G.Stokes, *On the theories of the internal friction of fluids in motion, and of the equilibrium and motion of elastic solids, 1849*, (read 1845), (From the *Transactions of the Cambridge Philosophical Society* Vol. VIII. p.287), Johnson Reprint Corporation, New York and London, 1966, *Mathematical and physical papers* **1**, 1966, 75-129, Cambridge.

Life in six Words. Print Version. In the 1920s, Ernest Hemingway bet ten dollars that he could write a complete story in just six words. He wrote: "For Sale: baby shoes, never worn." He won the bet. Now an online magazine is asking its readers to sum up their own lives in just six words. 'For Sale: baby shoes, never worn'. USEFUL LINKS.Â Smith, the American online magazine, has used the Hemingway anecdote to inspire its readers to write their life story in just six words, culminating in a book of the best contributions, entitled "Not Quite What I Was Planning". We interviewed magazine editor, Larry Smith, what made him think of the idea. Listen to the interview. Your six word memoirs. Already hundreds of you have emailed your own ideas about how to sum up a life in just six words. In his final hours as president, Donald J. Trump doled out pardons and commutations to dozens of people, including supporters, political figures, rappers and defendants in high-profile criminal cases. Mr. Trump named most of the recipients on a list released by the White House early Wednesday that included 73 pardons and 70 commutations.Â Alice Marie Johnson was serving life in a federal prison for a nonviolent drug conviction before her case was brought to Mr. Trumpâ€™s attention by the reality television star Kim Kardashian West. The presidentâ€™s decision to commute her sentence freed Ms. Johnson, who had been locked up in Alabama since 1996 on charges related to cocaine distribution and money laundering. George Floyd worked at the same local nightclub as the Minneapolis police officer who was shown on video kneeling on Floyd's neck as he said, "I can't breathe." Floyd, who died in police custody after his arrest on Monday, would occasionally provide security inside El Nuevo Rodeo club, according to former owner Maya Santamaria, who has since sold the club. Former officer Derek Chauvin, who has been identified as the man who knelt on Floyd's neck, would provide security outside the venue.Â Although Floyd and Chauvin worked at the nightclub at the same time, Santamaria said she doesn't believe they knew each other since they worked in different areas. Floyd was a sweet man with a big smile, she recalled. George Floyd.Courtesy photo. (Adverbial modifier) â€œHe entered the room, his hands in his pockets. â€œHow do you like the Army?â€ Mrs. Silburn asked.Â â€œThe register of his burial was signed by the clergyman, the clerk, the undertaker and chief mourner. Scrooge signed it. (Ch.Dickens).Â Their work on them by turns, and thinnâ€™d them tooâ€¦â€ (G.G.Byron) (There is nothing specially to arrest the readerâ€™s attention; each word is closely associated semantically with the following and preceding words in the enumeration.) His father, Simeon Poisson was a jurist. His mother was listed by his biographer and fellow scientist, Francois Arago, as mademoiselle Franchetere.[1] One story from his early years has it that a caretaker used to hang him from a nail so that she could perform errands without worrying about his straying about. In his later years, Poisson jokingly referred to his interest in pendulums as having begun at that stage of his life.[2]. At age 14, Poisson was placed in the care of an uncle who was a surgeon, in the hope that he would take up medicine as a career.[3] This was not to Poisson's lik...Â He was excused from his final exams upon taking up this appointment.